Journal of Physics: Conference Series

Demonstration of special relativity effects with specialized software

To cite this article: A V Kashnikov *et al* 2019 *J. Phys.: Conf. Ser.* **1348** 012092

Demonstration of special relativity effects with specialized software

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Abstract. A software development project designed to demonstrate some effects of the special theory of relativity (STR) is considered. The software visualizes a uniform relativistic motion of one inertial reference frame, associated with a rocket, relative to another inertial reference frame, associated with the ground. The main idea behind the visualization is to reduce the speed of light down to perceived value, such that relativistic time dilation and relativistic length contraction effects can be explored. The basic mathematical expressions describing the model are given, and the interface of the software is presented. It is expected that the developed software will help to provide intuition behind relativistic mechanics for students starting their immersion in this topic.

1. Introduction

The special theory of relativity (STR) [1, 2] is probably one of the most difficult topics to master in the general physics course [3]. one of the reasons for this is that it appears to be very difficult to imagine what is happening in the Lorentz transformation, mixing spatial and temporal coordinates, for the human brain habitually dealing with objects moving at velocities much lower than the speed of light. In the current paper, we consider a project devoted to helping students to develop their intuitive model of spacetime behavior.

The idea is to create a visual software, where the speed of light is artificially reduced down to perceived value (let say, length of a screen per second), and to show how what would the world look like in this case [4].

The work was inspired by the “Open relativity” project developed at Game Lab of Massachusetts Institute of Technology [5]. In this project, it was designed a 3d open-source toolkit to demonstrate the SRT effects within the Unity game engine. In our project, developed at Bauman Moscow State Technical University, we pursued less general goals and consider only a 1d case. At the same time, we focused our attention on pedagogical aspects rather than on entertainment features. Particularly, we concentrated on the demonstration of relativistic time dilation and relativistic length contraction effects. Our aim was to show the consistency of these effects being observed in different frames.

2. System under consideration

Consider two inertial reference frames (IRFs) K and K' with coordinate axes (x, y, z) and (x', y', z') correspondingly. In what follows we denote all the variables corresponding to K' (K) with (without) primes. Let the IRF K correspond to the "ground", while K' corresponds to a "rocket" moving with uniform velocity v directed along the x-axis (see Fig. 1). Let the axes x and x', y and y', z and z' be pairwise aligned, and at the initial time moment $t = t' = 0$ (related to both K and K') the points $(x = 0, y = 0, z = 0)$ and $(x' = 0, y' = 0, z' = 0)$ coincide.

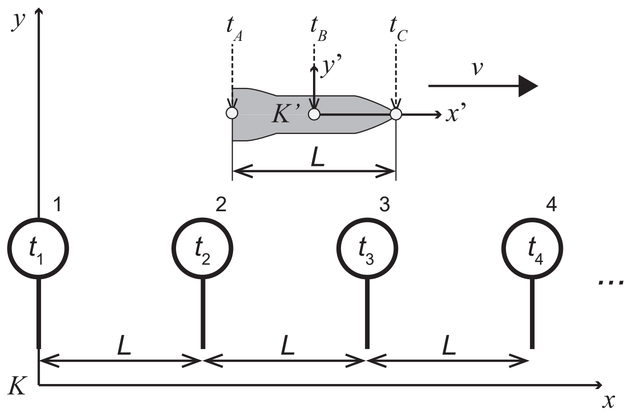

Figure 1. General scheme of the considered setup.

If the rocket's velocity v is much less than the speed of light c, then the spacetime coordinates (x, y, z, t) of an event in K are related to the spacetime coordinates (x', y', z', t') of the same event in K' by the Galilean transformation

$$x' = x - vt; \quad y' = y, \quad z' = z, \quad t' = t. \tag{1}$$

In the case, where the velocity v is comparable to light speed c, the Galilean transformations have to be replaced by the Lorentz transformations:

$$x' = \frac{x - vt}{\gamma}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - x\beta/c}{\gamma}. \tag{2}$$

In (2) the following standard notations were used:

$$\beta := v/c; \quad \gamma := \sqrt{1 - \beta^2}. \tag{3}$$

As might be expected, the transformation (2) turns into (1) with $\beta \to 0$. Note, that $y = y'$ and $z = z'$ in both transformations, so further we only follow the transformation of x (x') and t (t') coordinates.

The interesting consequences of the Lorentz transformation are the effects of length contraction and time dilation. It turns out that the length of a rocket from the viewpoint of an observer standing on the ground becomes shorter than the one for the observer inside the rocket. Furthermore, a time interval between two events happening at the same location in the rocket (say, in its middle) appears to be shorter than the corresponding time interval for the resting observer standing on the ground. These effects might appear to be counter-intuitive since the relative nature of the movement. Indeed, from the viewpoint of the observer in the rocket, it is the ground IRF that is moving, and so the length of objects standing on the ground has to contract, while the time periods between the ground events have to increase.

In order to study seeming contradictions, we introduce two sets of clocks. The first set consists of clocks fixed to the ground and spaced at a distance L from each other (see Fig. 1), where L

is a length of a rocket in its own IRF. We label these clocks with integers, and the coordinate x of the i^{th} clock in the IRF K is given by

$$x_i = L(i-1). \tag{4}$$

We set all the clocks to be synchronized with respect to IRF K, so their readings are given by

$$t_1 = t_2 = \ldots = t_i = \ldots = t. \tag{5}$$

The second set consists of three clocks A, B and C synchronized with respect to K' and are located at tail middle and nose of the rocket correspondingly. Their coordinates and readings with respect to IRF K' are given by

$$x'_A = -L/2, \quad x'_B = 0, \quad x'_C = L/2; \quad t'_A = t'_B = t'_C = t'. \tag{6}$$

Next, we discuss the behavior of the first set of clocks from the viewpoint of the observer in the rocket and the behavior of the second set of clocks from the viewpoint of the observer on the ground. In the case of $v \ll c$ it follows from the Galilean transformation (1) that

$$x_A = -L/2 + vt, \quad x_A = vt, \quad x_A = L/2 + vt, \quad t_A = t_B = t_C = t, \tag{7}$$

and

$$x'_i = (i-1)L - vt', \quad t'_i = t'. \tag{8}$$

These results seem quite natural and intuitive. However with the Lorentz transformations (2) we obtain

$$x_A = -\gamma\frac{L}{2} + vt, \quad x_B = vt, \quad x_C = \gamma\frac{L}{2} + vt, \quad t_A = \gamma t + \beta\frac{L}{2c}, \quad t_B = \gamma t, \quad t_C = \gamma t - \beta\frac{L}{2c} \tag{9}$$

and

$$x'_i = \gamma(i-1)L - vt, \quad t'_i = \gamma t' + \beta\frac{(i-1)L}{c}, \tag{10}$$

which looks very confusing. We note the following important points.

(i) The length contraction appears in both IRFs. Indeed,

$$x_C - x_A = \gamma L, \qquad x'_{i+1} - x'_i = \gamma L. \tag{11}$$

(ii) The time dilation also takes place in both IRFs, since the time reading can be written in the form

$$t_{A(B,C)} = \gamma t + \text{const}, \qquad t'_i = \gamma t' + \text{const}, \tag{12}$$

whereby const we denote quantities which do not depend on t or t'.

3. Demonstration of the STR effects with specialized software

In order to demonstrate the consistency of the Lorentz transformation, we developed a specialized software, which visualizes a system considered in the previous section. The software code was written in Python 3 with the Pygame library [6]. The source code is available at [7].

The initial menu of the program is shown in Fig. 2. It consists of the following options.

- **New experiment** launches the main part of the program described further.
- **Instruction** opens a PDF file with a detailed description of the program together with some problems proposed for solving within the framework of virtual laboratory work.

Figure 2. The initial menu screen.

- **Authors** give brief information about the developers.
- **Quit** closes the program window.

Selecting option **New experiment** opens the main program window shown in Fig. 3 and 4. The program can operate in two main modes: **Galilean transformation** and **Lorentz transformation**. The user can change the mode by pushing buttons in the lower right corner of the screen.

The main part of the screen is occupied by two frames. The upper frame corresponds to the IRF K and shows fixed on ground clocks with rocket moving to the right. The lower frame corresponds to the IRF K' and shows a fixed rocket with on ground clocks moving to the left. Two pairs of three clocks on the right side of the screen labeled with letters **A**, **B**, and **C** correspond to clocks in the tail, middle, and head of the rocket. Their readings are shown with respect to IRF K (at the top) and with respect to K' (at the bottom).

The scroll bar in the right allows the user to change the speed of the rocket expressed in a fraction of the speed of light c. For the demonstration purposes, it was fixed at the level of 300 pixels per second (the length of the resting rocket L is set to be 340 pixels).

The button **Start** launches the rocket movement. The button **Pause** freezes the motion. During the pause, the user can scroll the time using keyboard buttons left and right. The button **Stop** returns the system to the initial state $t = t' = 0$, $x_B = x_1$.

The main aim of the program is to demonstrate a consistency of the Lorentz transformation consequences given in Eqs.(9) and 10. The screenshot presented in Fig. 4 shows that the length contraction and time dilation take place in both IRFs indeed. However, despite the fact that in the upper frame the rocket is shorter than the distance between the pillars while in the bottom frame the distance between the pillars is shorter than the rocket, accurate comparison of the clocks readings in both frames shows that there are no contradictions.

If, say, at the moment when clocks B in the rocket pass over 5^{th} clock on the ground the corresponding clocks readings are $\hat{t}_B$ and $\hat{t}_2$ in the upper frame, then it can be checked that exactly the same situation appears is in the bottom frame. It would be impossible if only a transformation of either space or time coordinate was applied. But it is a consistency of Lorentz transformations with both of them which removes all the contradictions.

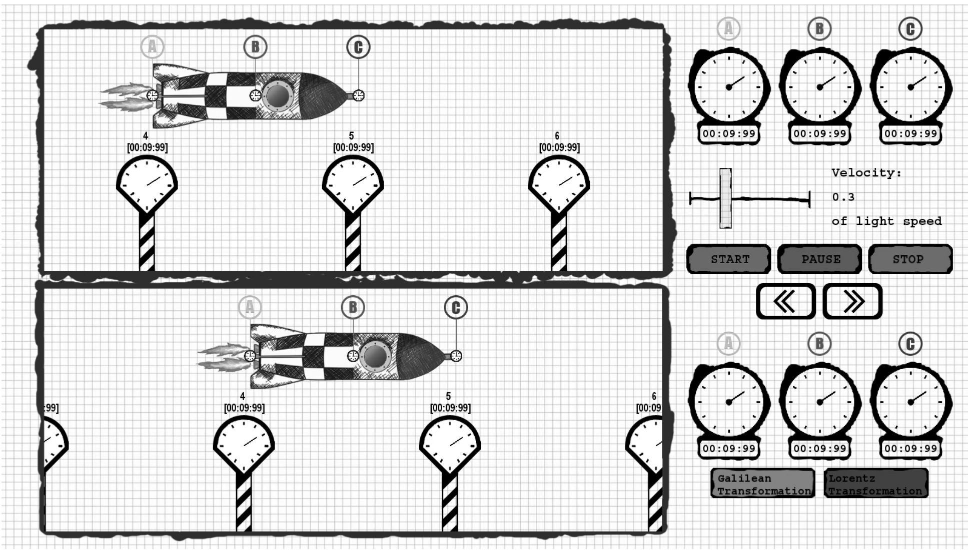

Figure 3. Program operation in the **Galilean transformation** mode.

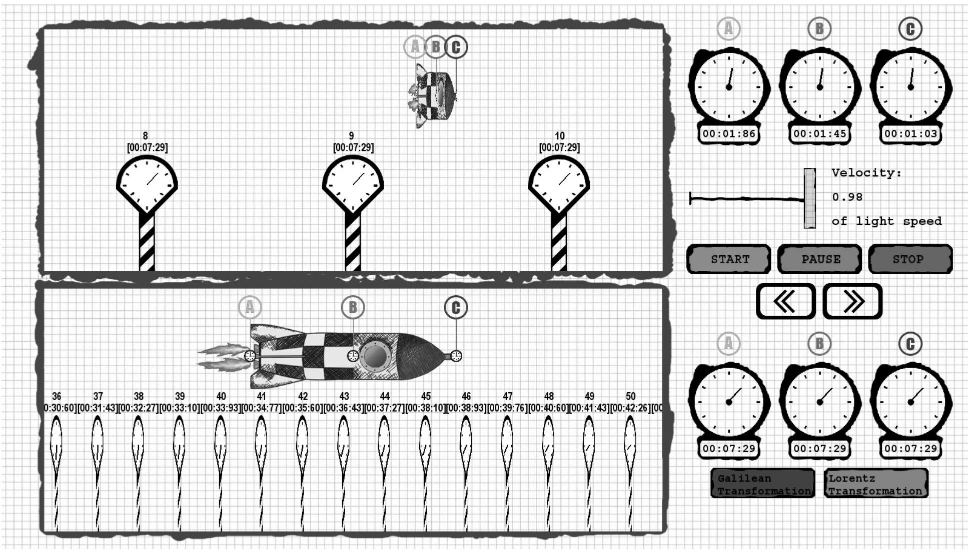

Figure 4. Program operation in the **Lorentz transformation** mode.

4. Conclusion and outlook

As a result of the work done it was created a specialized software for visualizing the basic effects of the SRT: relativistic time dilation and relativistic length contraction. The source code can be found at [7]. It is expected by the authors that the developed software will help to provide intuition behind relativistic mechanics for students starting their immersion in this topic.

As an addition to the software, it was developed a methodical manual with detailed mathematical description and self-evaluation problems. Based on this, virtual laboratory work was developed for undergraduate students at Bauman Moscow State Technical University.

Further, it is planned to move from 1d model to 2d model with the addition of the possibility of movement with acceleration. It will allow demonstrating the "twin paradox" together with time dilation and length contraction. Finally, it should be mentioned, that one of the main goals of the project is the introduction of interactive elements into the studying process and the use of software products not only in purely scientific but also in cognitive activity. From the authors' point of view, the employing of the game elements at the initial stage of studying can both increase interest in the particular subject for first-year students and improve motivation to do science for students of the high school.

Acknowledgments
The authors are deeply grateful to B. G. Skuybin and A. N. Morozov for the opportunity to develop the project in the framework of the Student Experimental Laboratory of Physics (SELPh).

References

[1] Einstein A 1905 *Ann. Phys.* **322**, 132
[2] Einstein A 1905 *Ann. Phys.* **322** 891
[3] Scherr R E 2007 *Am. J. Phys.* **75** 272
[4] Kortemeyer G, Fish J, Hacker J, Kienle J, Kobylarek A, Sigler M, Wierenga B, Cheu R, Kim E, Sherin Z, Sidhu S, and Tan P 2013 *Phys. Teach.* **51** 460
[5] Sherin Z W, Cheu R, Tan P, and Kortemeyer G 2016 *Am. J. Phys.* **84** 369
[6] Pygame library official website `www.pygame.org`
[7] The project GitHub repository `github.com/alexstux/special-theory-of-ralativity-lab`