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The invisibility of time dilation

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Abstract

Recently, the physics education community has taken a keen interest in modernising physics education. However, while topics in modern physics have great potential to engage students, these topics are abstract and hard-to-visualise. Therefore, many students hold mistaken pictures and misconceptions, which can impede learning. In this article, we expose a pervasive misconception about relativistic time dilation by presenting a thought experiment illustrating the difference between visual observation and coordinate measurement. We also point out how existing language can mislead and confuse students. In response, we describe an instructional approach that introduces world-maps, world-pictures and event-diagrams to clarify the applicability of key equations in special relativity for improved understanding. By unpacking 'the invisibility of time dilation' from the perspectives of both physics and pedagogy, we aim to help teachers provide clearer instruction.

Keywords: special relativity, visualisation, time dilation, physics education

Supplementary material for this article is available online

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1. Introduction

In the face of tremendous scientific and societal challenges, physicists and educators have called for educational shifts to infuse society with science-educated citizens who are able to make more informed decisions [1, 2]. Blandford and Thorne [1] note that a transformation of physics education can only happen successfully if physicists and educators work together as a community with common goals. In this context, they have called on the community to broaden, expand, and revamp physics education.

Einsteinian physics (quantum physics and relativity) has emerged as a domain in which physicists and educators have actively shared, developed, and discussed instructional

models and pedagogical approaches to improve educational practices (e.g. [3–8]). Einsteinian physics presents excellent opportunities to engage students and foster positive attitudes towards science because students perceive Einsteinian topics to be more relevant than topics of traditional school physics [9].

It is natural for those learning such abstract concepts to want 'to picture' what is going on. However, many Einsteinian phenomena are invisible to the naked eye and defy students' everyday experiences. Examples include curved spacetime [10], cosmic expansion [11, 12], and warped time [8].

In these hard-to-visualise learning domains, it is common for students (and educators) to hold mistaken 'pictures' and related misconceptions, which can impede learning. Unfortunately, in the case of special relativity, these erroneous pictures are often perpetuated by reliable, as well as unreliable sources. This is of significant concern since special relativity has gained popularity as an introduction to modern physics in upper-level secondary physics education [13].

In this article, we seek to support teachers by identifying a pervasive misconception related to 'picturing' time dilation. We then present an instructional approach to address the distinction between visual observation (what we see) and coordinate measurement (what we measure). While visual observation is a 'measurement' (astronomers definitely think so!), this is a useful way to distinguish the two.

In considering instructional aspects of time dilation, we extend an ongoing discussion that has focused on time dilation in the context of general relativity [8, 10, 14, 15].

2. Misconceptions in special relativity

Length contraction and time dilation are central to introductory courses on special relativity. Unfortunately, the language of special relativity is often misleading. For example, contrary to common belief, 'observers' will not always *see* a relatively moving rod as length contracted [16].

In the following, we briefly revisit this 'invisibility of length contraction' (which has been discussed extensively, for example [16–20]). We then turn to the 'invisibility of time dilation', which,

in relation to the Twin Paradox, has occasionally been alluded to [21, 22] and considered explicitly [23] but has not been discussed in a more general setting, identified as part of a pervasive misconception, or explored in terms of teaching strategies to address this misconception.

2.1. A known issue: the invisibility of length contraction

In 'The invisibility of length contraction' [17], Appell recently revisited a common misconception around relativistic visualisation.

For an object moving relative to us, of proper length ℓ, we will measure a shorter (contracted) length, L, given by the equation:

$$L = \ell/\gamma \tag{1}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ (the Lorentz factor) in which v is the relative speed of the object and c is the speed of light in a vacuum.

However, equation 1 does *not* describe 'what we see' but 'what we measure'. What we measure corresponds to a specific measuring procedure in which we simultaneously locate separate points on the moving objects in our coordinate system. In contrast, what we see also depends on the differing time light takes to reach our eyes from different parts of an object. Appell laments 'Some six decades after Penrose [18] and Terrell's [19] publications … many textbooks and science presenters still get length contraction wrong'.

The introduction of [24] provides a historical summary of the visualisation of relativistic objects and, along with the references, provides insight into the significant consideration given to this area.

2.2. A 'new' issue: the invisibility of time dilation

In terms of time dilation, one can ask questions such as: What would a passing relativistic spaceship look like? Would people be doing everything in slow motion and clocks appear to be running slow?

As with length contraction, many textbooks and science presenters get the answers wrong. The answers are similar to those for length contraction
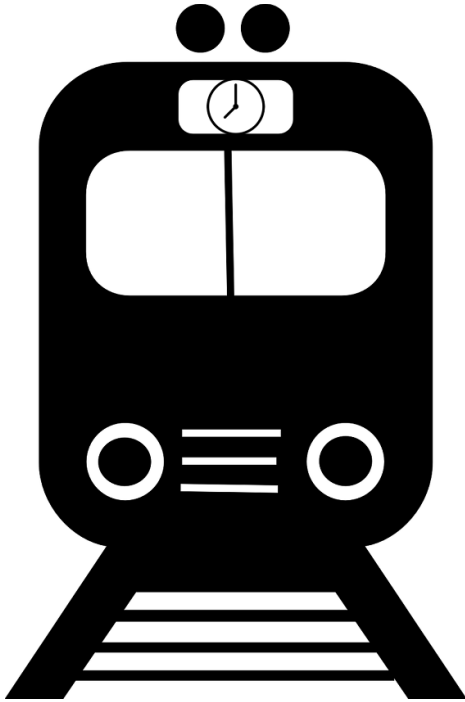
Figure 1. If a relativistic train is approaching you, would you see the clock running in slow motion?

and, as we will see later, have been 'hiding in plain sight'.

We would not be able to see a relativistic spaceship (or its occupants) even if we could arrange for one to pass. However, as with length contraction, an understanding of such scenarios can provide an intuitive model to understand relativistic phenomena better. It is also simply interesting and satisfies our natural curiosity! As such, educators (of all descriptions and using various media) often oblige this desire.

A typical example is a BBC YouTube video [25] in which a train with a clock on the front is approaching a station (figure 1). The video states that someone sitting on the platform would see the train's clock moving more slowly than their clock. Unfortunately, the BBC video's depiction is incorrect; they have made a common error.

Educators often present analogous examples such as: 'Your friend, Milly, is drinking coffee on a relativistic train approaching you. Due to time dilation, you see Milly drinking her coffee in slow-motion'. As the BBC video, this is incorrect.

Similar to Appell's discussion of length contraction [17], the misconception relates to confusing the time dilation equation:

$$\Delta t = \gamma \Delta \tau \tag{2}$$

with what we 'see' (where $\Delta \tau$ is the proper time interval between two events, Δt is the dilated time interval, and γ is defined as for length contraction).

As with length contraction, the time dilation equation does *not* describe 'what we see'. It describes a particular measurement procedure ('what we measure').

We illustrate this with the following example and then discuss instructional approaches to address such misconceptions.

3. Example: time dilation and a relativistic train

Imagine you are watching a large clock on the front of an approaching train (as in the BBC video, and figure 1). A friend, Charlie, is also in your inertial reference frame. She is standing further up the platform so the train will pass her first.

We have two events that we have depicted using a Minkowski diagram as well as the equivalent event-diagram (figure 2):

- Event 1—the front of the train is opposite Charlie.
- Event 2—the front of the train is opposite You.

(We will discuss event-diagrams later. For now, just think of them as a pictorial representation of the Minkowski diagram.)

To investigate the time interval between the events, we refer to your inertial reference frame as S (with variables x, t and time interval Δt). The train's reference frame as S' (with x', t' and time interval $\Delta t'$). Figure 2 shows the situation from the perspective of S (your reference frame).

For Event 1 we have chosen $t = t' = 0$ and $x = x' = 0$ (the axes of the coordinate systems coincide, and we have chosen Event 1 to be at the origin, for simplicity).

To help the reader, we make the scenario more concrete using some simple example values. We say:
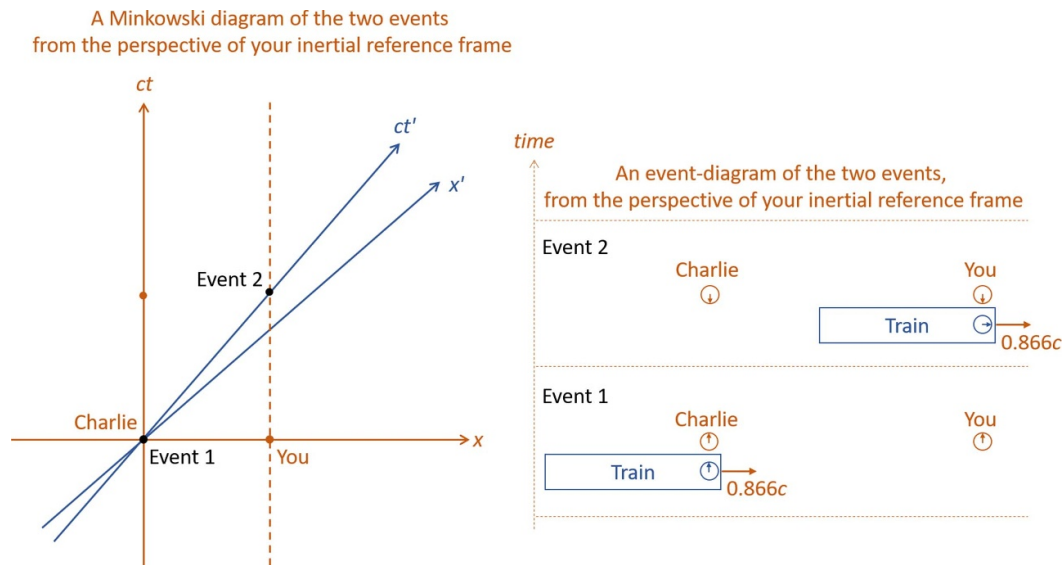
Figure 2. A Minkowski diagram and an event-diagram illustrating the thought experiment of a relativistic train passing Charlie and you standing on the platform.

- The train is traveling at 0.866 c ($\gamma = 2$).
- In your frame, Event 2 occurs at $t = 2$, so $\Delta t = 2$.

The clock on the front of the train, and Charlie's, are set to zero at Event 1. Your clock is synchronised with Charlie's, in your reference frame. So, when the train is opposite you (Event 2) your clock will be reading 2 s. However, by equation 2 (time dilation) $\Delta t' = 1$. So, when the train's clock is opposite you (Event 2) it is only reading 1 s.

The event-diagram in figure 2 shows all of this: for Event 1, all the clocks are pointing up (indicating time zero). At event 2, your clock (and Charlie's) are pointing down, indicating 2 s later, whereas the clock on the front of the train is pointing to the right—only half the distance your clock has moved, indicating half the time, just 1 s.

We have $\Delta t > \Delta t'$ (time dilation), so one could say 'time is running slower in the reference frame of the train'. However, this can lead to multiple misconceptions [26] such as: thinking that passengers on the train will feel time running slower (they would not) and that if you watched the clock on the front of the train, you would see it 'running slow' (you would not).

3.1. So, what would you see?

In your reference frame, the distance between you and Charlie is (distance = speed × time):

$$0.866 \times 2 = 1.732 \text{ light seconds.}$$

Meaning light from Event 1 takes 1.732 s to reach you. So, you see Event 1 when your clock is reading 1.732 s, *not* 0 (figure 3).

Between seeing Event 1 and Event 2, you see the train's clock move from 0 to 1 s, but your clock only moves from 1.732 to 2 s. The train's clock moves 1 s while your clock only moves 2–1.732 = 0.268 s. The train's clock moves further: you see the clock on the front of the train moving faster! Similarly, you would see your friend on the train, Milly, drinking her coffee very rapidly.

So, generally, you do *not* see time dilation. What you see is due to a combination of time dilation and the finite speed of light.

The dilated time, Δt, is what you and Charlie 'measure': the difference between the time coordinates of the two events in your reference frame, S. It represents the comparative reading on two clocks, *at the events*, in your reference frame (where the clocks are synchronised in your reference frame). The dilated time is *not* what you

A Minkowski diagram indicating when you see Event 1
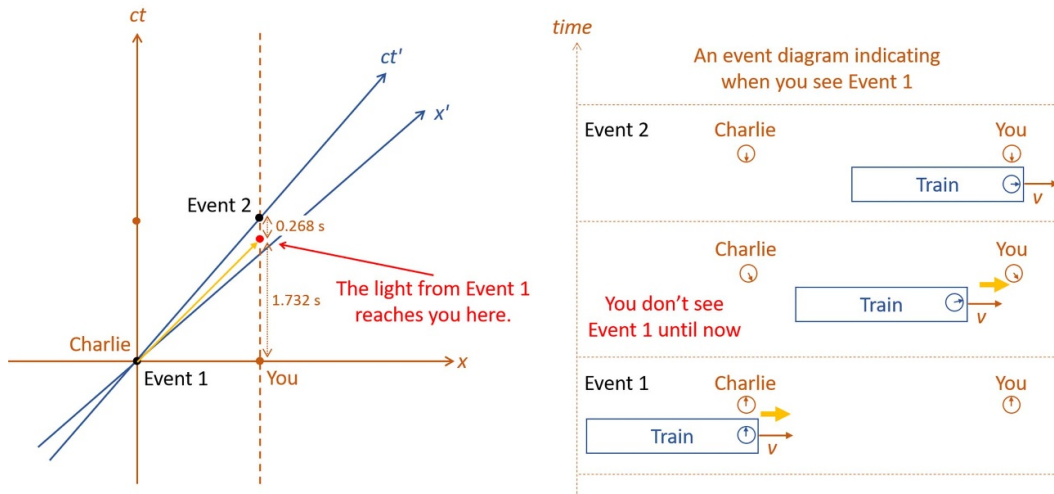
Figure 3. A Minkowski diagram and corresponding event-diagram indicating what you would see if a relativistic train approached you. Contrary to common belief, you would see the clock on the front of the train moving faster.

see on your clock compared with what you see on the approaching clock.

Also, we have discussed what *you* would see. What Charlie would see (even though she is in the same reference frame) would be different.

3.2. The general case

In general, if $\Delta t'$ is the time an approaching clock records between events (like the train's clock) then the time you see between the events, Δt_S (S for see), is:

$$\Delta t_S = \kappa \Delta t' \qquad (3)$$

where $\kappa = \sqrt{(1-\beta)/(1+\beta)}$ and $\beta = v/c$.

Setting $\Delta t' = 1$ and $\beta = 0.866$, you can verify the answer for our example ($\Delta t_S = 0.268$ s). You could ask students to derive this formula—setting up the variables can be tricky, but the algebra is relatively straightforward. We provide a solution in the accompanying materials[4].

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[4] You can download the solution as an appendix (available online at: stacks.iop.org/PED/56/025011/mmedia) to this article.

3.3. An 'old friend'

The expression for κ should look familiar. It is the 'relativistic Doppler effect'. However, be careful with signs when comparing this to other references. We are always taking β positive, whereas some references define β as negative for an approaching (or receding) source, and/or define their factor equivalent to $1/\kappa$.

Imagine light emitted from the front of the train with frequency f_E (E for emitted). You would see its frequency as f_S (S for see) such that:

$$f_S = f_E/\kappa. \qquad (4)$$

As $\kappa < 1$, the frequency you see for an approaching light is greater than the emitted frequency—the classic blueshift encountered in astronomy.

Similarly, the time and frequency for a receding train (assuming a clock and light on the back) are:

$$\Delta t_S = \Delta t'/\kappa \qquad (5)$$

$$f_S = \kappa f_E. \qquad (6)$$

The clock would appear slower (but slower than just time dilation) and the light would have a lower frequency (redshift).

The relativistic Doppler effect is well-known and has been included in relativistic visualisations (e.g. [24]). Hence why we said visualisation of time dilation has been 'hiding in plain sight'. It is usually derived by taking the classical Doppler effect and correcting for time dilation. Whereas the derivation in the accompanying material is from 'first principles'.

Interestingly Bondi's *k*-calculus [27] (a much less common approach to teaching special relativity) takes the Doppler effect (effectively κ) as a starting point and from it derives time dilation, length contraction and the Lorentz transformations. While, similarly, this approach does not explicitly address the misconception between world-maps and world-pictures (see later), using it, one could equally apply the strategy of emphasising the distinction between world-maps and world-pictures.

The Doppler effect is rarely explicitly linked with how one 'sees' time dilation except, for example, in a limited way such as Schild [23] in relation to the Twin paradox and, in a more general sense, one sentence in Rindler [28] where, in discussing the Doppler effect, he says 'Note that the above argument and formula apply equally well to the visually observed frequency ν of a moving clock of proper frequency ν_0'.

Note: In 1D the relativistic Doppler effect also relates directly to the train's length. However, think carefully in relating this to the Doppler-shifted wavelength of light. For an approaching train, the wavelength would appear shorter (multiply by κ) but the train would appear longer (divide by κ)! To explain this is an exercise for the reader or the reader's students[5].

4. Instructional strategy: world-map and world-picture

The misconceptions relating to length contraction and time dilation both stem from confusing what an observer sees vs what they measure. In the Instructor Guide to [29], Knight states 'Although there's no research on the issue, I am inclined to believe that the traditional relativity term *observer*
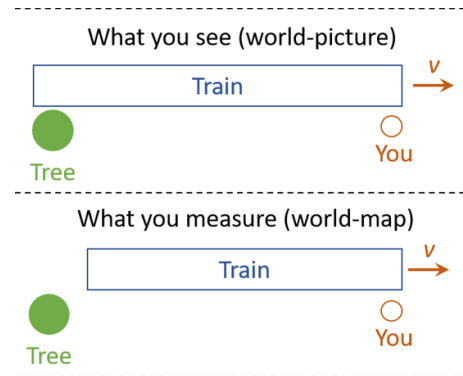
Figure 4. World-pictures and world-maps help students distinguish what you see from what you measure when dealing with relativistic phenomena such as length contraction and time dilation.

contributes to the difficulty students have distinguishing the perception of an event from the occurrence of an event. This term brings with it an implication that the perception of an event (i.e. the *observation*) is what is important'.

Research in quantum physics education suggests that secondary school students do tend to conceptualise 'observation' as seeing rather than measuring [30], providing further evidence that conflating the two notions is a likely source of misconceptions.

To attempt to counter this issue, Knight replaces 'observer' with 'experimenter'. A potentially useful strategy applicable across relativity and quantum physics—although the pervasiveness of the term 'observer' might make the adoption of this change an uphill battle. We suggest an additional strategy, aimed at the particular issue under discussion.

We introduce useful but under-utilised terminology, coined by Rindler [31]: world-picture (what you see) and world-map (what you measure). These terms can be used, for example, to distinguish 'what you see' from 'what you measure' in relation to a passing train (figure 4).

4.1. World-picture

A world-picture is a diagram from a particular point in space. Figure 4 shows the world-picture from where you are standing. It illustrates the

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[5] You can download the solution as an appendix to this article.

image formed on your retina (or a digital camera sensor).

The world-picture shows the effect of light taking time to reach you from other locations. Every part of the image is from a different time in your reference frame. For example, you see the front of the train in your 'now' but the back of the train from some time in your past. So, the back appears further away, and hence the train appears longer. Note: the train appears longer because this effect is greater than length contraction.

4.2. World-map

A world-map illustrates the situation from a particular inertial reference frame (rather than a point in space). In 1D, as here, it shows everything that happens at one time (simultaneously) *in that inertial reference frame.* It shows the reference frames' coordinates for objects and events, all the *x*-values for one *t*-value. (A Minkowski diagram is a world-map that shows multiple *t*-values).

The train appears shorter than in the world-picture, and this *does* represent the contracted length of the train (the proper length being somewhere between the two).

4.3. Instructional use in the classroom

The distinction between a world-map and a world-picture can be crucial in non-relativistic situations. For example, astronomical observations can include objects from significantly different times. However, the importance to the current discussion is that world-maps depict the equations of length contraction and time dilation. A world-picture *does not*.

The terms provide useful and efficient terminology to describe the confusion inherent in many expositions of special relativity: people confuse world-pictures with world-maps. However, we would suggest adopting this terminology not just for remediation (addressing existing confusion), but to try and avoid such confusion in the first place.

Before introducing relativistic phenomena, we suggest teachers discuss world-maps and world-pictures. These terms then provide an efficient, simple way to describe, emphasise, and

remind students of the applicability of these equations. For example, the Doppler effect applies to world-pictures; length contraction and time dilation apply to world-maps. Also, introducing Minkowski diagrams, teachers can emphasise these are world-maps.

In figures 2 and 3, we used event-diagrams (one or more world-maps) to complement the more abstract Minkowski representation. The event-diagram is a pictorial representation of horizontal slices of interest from a Minkowski diagram. Event-diagrams are similar to diagrams used in textbooks. However, there is a set of rules around their use. These rules encode key concepts about special relativity that are often not sufficiently emphasised. The diagrams, in turn, provide a useful tool to introduce these concepts. The rules and diagrams are synergetic.

World-pictures, world-maps, and event-diagrams can serve multiple instructional purposes:

(a) They provide a visual tool to support the introduction of fundamental relativistic concepts with a clear distinction between visual observations and coordinate measurements.
(b) They are a useful stepping-stone to introducing the more abstract Minkowski diagram.
(c) They can complement Minkowski diagrams.

Event-diagrams as formal, pictorial world-maps were developed by Hughes (to be described in a future publication). They have been used in teaching (at the university and high school level) by Hughes, since 2014[6]. Similar diagrams have been described in [13, 32].

5. Summary

Since relativistic phenomena are hard-to-visualise, students often hold mistaken 'pictures'. Specifically, confusion between 'what we see' and 'what we measure' can lead to persistent misconceptions around length contraction and time dilation,

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[6] Level 98 is developing an online course for High School Physics which includes a section on special relativity. This section provides a pedagogical demonstration of incorporating the terms world-map and world-picture and using event-diagrams (along with other innovations).

as well as more general confusion about special relativity. The history of this confusion, related to length contraction, indicates the issue is pervasive, as well as tricky to rectify. The lack of realisation that there is a similar issue around time dilation suggests the problem is subtle.

In this article, we expose this pervasive misconception about time dilation. We present a familiar thought experiment demonstrating 'the invisibility of time dilation' and show how the language of special relativity can mislead students (and educators). In response, we describe an approach that introduces world-maps, world-pictures, and event-diagrams as useful instructional tools. By unpacking the invisibility of time dilation from the perspective of both the physics and the pedagogy, we aim to support teachers who wish to bring one of the most striking features of Einstein's theory of relativity into their classrooms. We hope future work will investigate the efficacy of this approach.

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