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Department of Physics and Astronomy
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Relativistic Visualizations

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Abstract

Special relativity is an area of physics that is abstract and consists of concepts and effects that is hard to link to everyday experience. From a learning perspective, that is problematic as we form new knowledge by linking it to old experiences. One approach to experience a relativistic environment is through computer simulations. MIT game lab has developed the game “A Slower Speed of Light” where the user slows down the speed of light in discrete steps. That allows the user to experience what the surroundings would look like if we were traveling at a relativistic speed. As the user does that, he/she will experience that certain visual effects do not appear as the course literature describes it. In this project, we study how visual effects of special relativity are perceived by students by letting them play the game and solve an assignment. In the assignment, there was one problem where they were instructed to identify two relativistic effects and describe the physics behind them. The project aims to find an answer to what students while playing the game think they see and how they relate the visual effects to quantities arrived by applying the Lorentz transformation. The project also investigates whether the game can be used productively in teaching. The findings of project show that it is easier for students to describe visual effects where observation match with what the students actually see. We have also *found* that the students' perception of visual effects is that they are consistent with the predictions of the Lorentz transformation as they try to force their observations onto the theory. That despite the fact that the game clearly shows that it is not the case. The game can be used productively by a teacher when lecturing e.g. relativistic optics.

Sammanfattning (Swedish)

Speciell relativitetsteori är ett område inom fysiken som är abstrakt och består av begrepp och effekter som är svåra att länka till vardagslivet. Från ett lärandeperspektiv blir det problematiskt då vi bildar ny kunskap genom att länka det till gamla erfarenheter. Ett sätt att få uppleva en relativistisk miljö är genom datorsimuleringar. MIT game lab har utvecklat spelet “A Slower Speed of Light” där användaren stegvis saktar ner ljusets hastighet. Detta gör det möjligt för användaren att uppleva hur omgivningen skulle se ut om vi färdades i en relativistisk hastighet. När användaren färdas relativistiskt kommer han/hon uppleva att vissa visuella effekter inte stämmer överens med beskrivningar från kurslitteratur. I detta projekt studerar vi hur visuella effekter av den speciella relativitetsteorin uppfattas av studenter genom att låta dem spela spelet och lösa en inlämningsuppgift. I inlämningsuppgiften fanns ett problem där de skulle identifiera två relativistiska effekter och beskriva fysiken bakom dessa. Projektet försöker att besvara vilka effekter studenter tror att de ser när de spelar spelet men också hur de relaterar de visuella effekterna till värden som förutsägs genom Lorentz transformationen. Projektet undersöker också om spelet kan användas produktivt i undervisning. Projektets observationer visar att studenterna har lättare att beskriva visuella effekter där observation stämmer överens med vad studenterna faktiskt ser. Vi har också *noterat* att studenternas föreställning om visuella effekter är att de stämmer överens med Lorentztransformationens förutsägelser då de försöker tvinga sina observationer att överensstämma med teorin trots att spelet tydligt visar att det inte är fallet. Spelet kan användas produktivt av en lärare vid föreläsning av t.ex. relativistisk optik.

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Chapter 1

1 Introduction

1.1 Introduction

As a future physics teacher, I want to identify potential challenges that students are likely to encounter in physics and at the same time broaden my understanding of the subject. One area where difficulties often occur according to my own experience is in special relativity. Special relativity was published by Einstein in 1905 and it replaced ideas of space and time that the Newtonian mechanics described.

In education, it is common to use special relativity as an introduction to modern physics of which the purpose is not only learning the particular topic but to develop their abstract thinking. But as students get exposed to abstract topics it occurs that they tend to focus on manipulating different equations instead of learning the concepts (McGrath 2010). From my experience, that procedure might be useful to pass an exam. But it is not as effective to understand the content. An explanation could be that the math in special relativity is pretty straight forward but also because it consists of concepts that are counterintuitive and the effects of special relativity are difficult for students to directly experience. That can be problematic from a learning perspective.[1] Consequences in special relativity are that you cannot travel faster than the speed of light, distances gets contracted and the clock is running slower. Suppose a person is traveling at speed v relative to you. If you look at that persons watch you would notice that it runs slower than your own. In high-school, students are supposed to master those consequences. At my internship, a high-school student asked a question about length contraction:

"*So, if I travel close to the speed of light, would I be able to see a city that is 100 km away?"*

When a teacher answers this question, it must be done carefully. The student actually asks two questions. The first one is if the students' interpretation of the mathematical models of special relativity is correct. The second question is if one would actually experience a visual effect like the one described above when traveling close to the speed of light. The short answer to that is that one would *not.*

There is a difference between see an effect and observe it. In this study, we make a distinction between *see* and *observation*. If I see the shape of an object that is traveling, it is referred to photons from different parts hitting the eye simultaneously in *my* frame of reference. An observation of its shape is referred to simultaneous measurements of two points of the object. That is the photons that simultaneously *left the surface* of the object. Hence the speed of light is finite, the light that hit your eye (see) was *not* the photons emitted simultaneously from the object (observation). We illustrate the difference by following example. Observing the length of a train in motion can be done by recording when the edges passed a detector. Suppose we

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[1] Haglund (2013) summarizes a large body of literature on educational constructivism, experiential learning and the use of analogies in education, which suggests that students build their understanding of new phenomena/information on their own previous experiences and knowledge.

know its velocity, then we can calculate its length by multiplying the velocity with the time interval. If the speed was relativistic, our observation says that the train is shorter compared to when at rest. The numerical value we calculate would match with the prediction of the Lorentz transformation. But this procedure is not the same as see. When we see an event, it means that our eye is getting hit by photons simultaneously. But light has a finite speed and therefore, looking at different parts of the train that have different distances to the eye, the photons arriving at our eye simultaneously were emitted from the train at different times. This consideration was taken up and developed by Lampa (1924). He stated that what you measure (a simultaneous measurement of the different parts of the object.) is different from what you see and it took decades before Terrell (1959) and Penrose (1959) further developed that idea. They concluded that the length contraction can only be seen in a special case where the object must be seen perpendicular to its direction of motion. In special relativity, there are also effects where see and observation match. One example is the relativistic Doppler effect which describes how properties of light e.g. frequency is altered by motion. If you look at an object you would see a color and observing the wavelength by measuring it with a spectrometer, it would give a value of the wavelength that is consistent with the perceived color. Studying the properties of light e.g. its color only requires an observation of one photon. When we study shapes of objects, we must determine the positions of at least two photons at the same "emission time". That aspect is not necessary studying the color of the photons.

One tool to show the difference above to students is to expose them to a relativistic environment using computer simulations. MIT Game Lab developed a game engine called "*A slower Speed of Light"* which is a part of the *Openrelativity project* (Sherin 2016). In the game, the player gets an opportunity to see what a relativistic environment would look like. It achieves this by simulating an environment in which the speed of light is successively slowed down to be similar to the speed of the game-user's own speed in the game. It does so by taking the aspects that Lampa, Penrose and Terrell emphasized into account (observation not always what we see).

The game visualizes relativistic kinematics and relativistic optics. While the game has been constructed and released as an open source, there are few research publications on applying it in education. The idea of constructing the game was to stimulate students' intuition about special relativity (Kortemeyer 2013). There are several cases where visualization of concepts from different domains in physics have been effective, including PhET simulations e.g. McKagan (2008) and Kohnle (2015). Therefore, it is of interest to study if the game "A Slower Speed of Light" could develop students' intuitions about special relativity. To start the inquiry into this topic, I am interested in students' perceptions of the differences in the values of physics quantities (i.e. object length, incoming light color) as they are calculated using Lorentz transformations, and the visual perception of those same quantities that one can experience when one travels close to the speed of light on the other.

1.2 Purpose

The purpose of this project was to study how the students perceive visual effects they are exposed to in a simulated relativistic environment. I am especially interested in how they relate the observed effects to theoretical values of physics quantities arrived at by applying Lorentz transformations.

1.3 Research Questions

The project aims to find an answer to following questions:

- What relativistic visual effects do students report to have noticed while playing the educational computer game “A slower speed of light”?

- How students relate (a) the observed visual effects and (b) values of physics quantities arrived at by applying Lorentz transformations?

- Can this computer game be used productively for physics teaching? If so, in what way and for what purpose?

Chapter 2

2 Theoretical Framework

The theoretical framework behind this study has been divided into three subcategories: physics, visual effects and educational.

2.1. Physics

This subcategory summarizes the relevant aspects of special relativity

2.1.1 Galilean perspective

In order to understand why we need special relativity we must understand the complications in the Galilean perspective. Discussing an event, four coordinates are required. First, we need to consider its position which is the three spatial coordinates (x, y, z). We also need to consider when in time. These four quantities can be illustrated as a point in space-time,

$$(t, x, y, z)$$

where t represents the time and x, y, z represents the spatial coordinates of the event. This allows us to refer to any event as a point in a 4-dimensional coordinate system that we call a reference frame. Figure 1 is a geometric representation of one reference frame. Note that it only has 2 spatial directions (xy-plane) instead of 3.

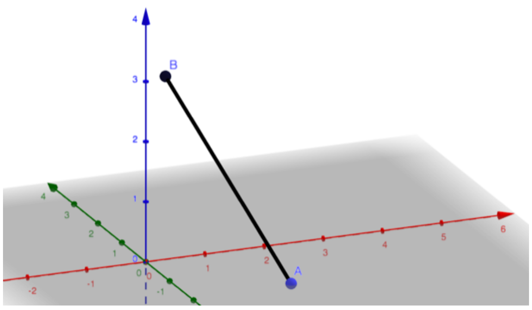


Figure 1. Spacetime diagram in 2-dimensional space.

The reference frames that are of importance in this project are the inertial ones. An inertial reference frame is a system that have following properties. The first one is that it has Euclidean spatial components which implies that every spatial component satisfies all the axioms of Euclidean geometry. The second is that Newton’s first law is fulfilled:

“If the net force acting on a body is zero then it is at rest or traveling in constant velocity.”

(1) $$\sum_{i=1}^{N} \vec{F_i} = 0$$

The third property that an inertial frame has is that there is a universal time. That means wherever in space you place clock it shows the same time. That leads to the question if one observer sees an event, how would an observer in motion see the same event. Suppose one observer is riding a train at constant velocity v and a different observer waiting at the platform. We refer to the stationary observer as **S** and to the observer on the train as $\mathbf{S'}$. How are these reference frames related? From the definition of an inertial reference frame the time of the observers is the same.

(2) $$t = t'$$

None of the observers is accelerating, therefore the transformation between the positions of the observers are related by:

(3) $$x' = x - vt$$

(4) $$x = x' + vt'$$

In (3) and (4) v is the velocity of the train. We can use those equations to find the transformation between the two observers velocity by taking the derivative of (3).

(5) $$u' = \frac{dx'}{dt'} = \frac{d(x-vt)}{dt'} = \{t = t'\} = \frac{dx}{dt} - v\frac{dt}{dt} = u - v$$

Now since observer S was stationary we know that velocity $u = 0$. Then in the frame of S', S is perceived to recede at a velocity of $(-v)$.

2.1.2 Michelson-Morley experiment

In 1887 Michelson and Morley performed an experiment that gave a result which became important. As a mechanical wave propagates, it needs a medium. When we hear an event, that is particles/molecules in the air vibrating. It is the vibrations that reach our ears. When waves in water is propagating it is the same principle. Therefore, it was assumed that electromagnetic waves also needed a medium to propagate. Since we can see the stars and galaxies, if there is a medium it must be spread through the universe. It was referred to as the *ether*. Michelson and Morley tried to estimate the earth´s velocity relative to the ether. In figure 2, the experimental setup was a light source emitting light to a half-silvered mirror that split the beam into two arms of equal length perpendicular to each other. Each arm had a mirror at its end that reflected the light back. As the two separated waves were recombined into one, they form an interference pattern on a screen. If there is an ether traveling through the earth in any direction then it should have an impact on the two waves. Michelson and Morley made a prediction that the changes in the paths of the light would give rise to phase shift between them. That would affect the interference pattern. The phase shift of the two separated waves were predicted to interfere destructively. However, that was not what Michelson and Morley observed. The two separated waves appeared to interfere constructively. Which meant that the waves were unaffected by the ether.

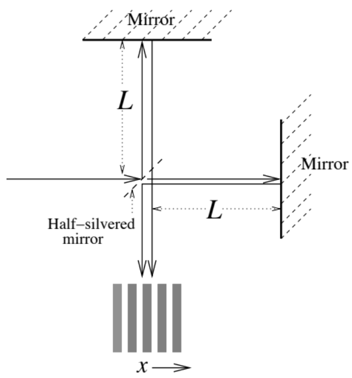


Figure 2: Setup of the Michelson Morley experiment. Credit Minahan (2011).

Michelson and Morley were aware that since the earth is orbiting the sun there was a possibility that the Earths velocity relative to the ether would be zero. In figure 3, at spring the velocity of the earth is in the same direction as the ether. Therefore, if the same experiment was done a half year later then the relative speed would be different (Minahan 2011, textbook describing the Michelson-Morley experiment). However, repeating the experiment, Michelson and Morley that did not observe any difference. The result of the experiment was the same. It seemed that the earth was at rest with respect to the ether. However, it provided an important conclusion. That measuring the speed of light is independent of the earths velocity.

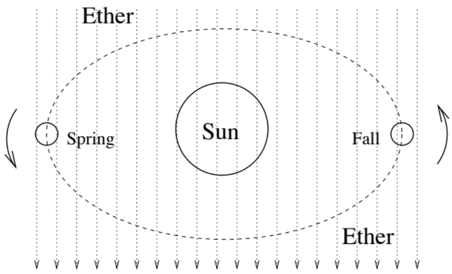


Figure 3. The earth is orbiting the sun implying that the Earths relative velocity to the ether change in time. Credit Minahan (2011).

Note that opposite directions of the Earth velocity mean they were in different reference frames. The measurement of the speed of light seemed to give the same result regardless of which inertial frame its being measured from. This became important to the two axioms that Einstein made in 1905. He stated the following (Rindler 1991, page 7, 8):

i) *The laws of physics are identical in all inertial frames.*

ii) *There exists an inertial frame in which light signals in vacuum always travel rectilinearly at constant speed c, in all directions, independently of the motion of the source.*

A consequence of ii) is that c is constant in every inertial frame. The first axiom is not problematic in Newtonian mechanics. It is the second axiom that is problematic. Measuring the speed of light between reference frames applying a Galilean transformation would lead to the conclusion that one of them is not an inertial frame.

A concrete example of a complication is a person on a train that travels at speed v in the x-direction. Suppose the person turns on a flashlight in same direction. If you measure the speed of the emitted photons, the Galilean transformation (5) will not give you the speed that the photon has. In the Galilean perspective, you would predict the following:

$$u = c + v \tag{6}$$

This violates the 2nd axiom. It actually does not matter in which direction the person on the train would point the flashlight. You would always measure the speed of light to be c.

2.1.3 Discovery of special relativity

The theory of special relativity was published by Einstein in 1905, his theory replaced the ideas of space and time that the Newtonian mechanics described. Special relativity describes how time and space are altered by motion, those effects is of importance to consider in physics when an object has a velocity that is a large fraction of the speed of light. That means, motion situations from everyday life e.g. flying at an airplane or driving on the highway you will not clearly experience the effects in special relativity. If the intention is to describe an event from everyday experience the Newtonian-Galilean framework is enough. The differences that the special relativity predicts are negligible in those cases. However, if we are describing the motion of a particle in an accelerator or certain properties of matter, special relativity is relevant. If it is not taken into account we get contradictions. An important limitation of special relativity is that it assumes no gravity. Gravity extends special relativity into general relativity. Before discussing if we would see the effects we will examine how they are described in textbooks. I will give a conceptual summary of relativistic effects. Derivations will be provided at a later at section 2.1.4-2.1.5.

2.1.3.1 *Lorentz transformation*

The Lorentz transformations describe how different observers observe the same event. In order to describe an event, some parameters are required. We need to describe where in space an event occurs, likewise we need to describe when the event occurs. Therefore, an event is described by a spatial vector $\vec{x} = (x_1, x_2, x_3)$ and a time coordinate t. Combining them, an event is presented as a point in spacetime that have 3 spatial directions. That event is observed as $(t, \vec{x})$ from an observer in inertial frame S, while an observer from a different inertial frame S', the event has coordinates $(t', \vec{x}')$. The algebraic path between these two frames is the Lorentz transformation (Holst. 2006, course literature).

2.1.3.2 *Time dilation / Length contraction*

Two effects that follow from the Lorentz transformation are length contraction and time dilation. Suppose an observer in motion wears a watch and holds a meter stick. And suppose a stationary observer do the same. If they observe the accessories of each other, then they would say that the clock is running slow and that the meter stick is shorter. Rindler (1991) call these effects *velocity perspective* as they are greater when the velocity approaches the speed of light. These two effects do not occur in Newtonian mechanics.

2.1.3.3 *Relativistic Doppler effect*

The Doppler effect describes how properties of waves are altered by motion. In Newtonian mechanics, the Doppler effect is noticeable when an ambulance is approaching you. The frequency is higher compared to when it is driving away. What happens is that the oscillating period decreases which means that the frequency increases. The relativistic Doppler effect becomes relevant discussing electromagnetic waves. An electromagnetic wave has properties as frequency and like the mechanical wave it changes if the observed object is traveling at a speed v. The color changes (follows the rainbow). If an observer is approaching an object then the object appears more to the blue spectra or more to the red if it is receding. However, the changes in colors requires relativistic speeds. What makes the non-relativistic and the relativistic Doppler effect different is that the relativistic case there is not an absolute time. Time dilation must be included in the relativistic case. The stationary observer (S) would say that his clock is running faster than the observer (S') in motion. Time running faster implies that the oscillating period appear shorter (increased frequency).

2.1.4 Derivation of Lorentz transformation

The question is how can we modify the Galilean transformation to a more appropriate one and at the same time, keeping properties such as the linearity. Suppose that we have an observer at rest (S) and a moving observer (S´) with constant velocity v in the x-direction. We introduce an extra factor γ and include Einstein's second postulate. The Galilean transformation equations is modified as follows:

(7) $$x' = \gamma(x - vt) \quad x = \gamma(x' + vt')$$

Suppose a light source is emitting photons from the origin, from Einstein's second postulate, following assumptions can be made:

(8) $$\Delta x = c\Delta t \quad \Delta x' = c\Delta t'$$

Now we solve for γ by plugging in the results from equation (7) into equation (8) and get following:

(9) $$c\Delta t' = \gamma(c\Delta t - v\Delta t) = \gamma(c - v)\,\Delta t$$

(10) $$c\Delta t = \gamma(c\Delta t' + v\Delta t') = \gamma(c + v)\,\Delta t'$$

The constant γ is derived by multiplying equations (9) and (10)

$$c^2\Delta t\Delta t' = \gamma^2(c - v)(c + v)\Delta t\Delta t' = \gamma^2(c^2 - v^2)\Delta t\Delta t' \leftrightarrow$$
$$\gamma^2 = \frac{c^2}{c^2 - v^2} = \frac{c^2}{c^2\left(1 - \left(\frac{v}{c}\right)^2\right)} = \frac{1}{1 - \left(\frac{v}{c}\right)^2} \leftrightarrow$$

(11) $$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Equation (11) is known as the *Lorentz factor*. The Lorentz factor $\gamma \in [1, \infty)$ as $(v/c) = \beta \in [0,1)$ which is illustrated in figure 4. Insert this result into equation (7) to get the space coordinate and time coordinate (by similar substitution) in a different reference frame:

$$x' = \gamma(x - vt) \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x = \gamma(x' + vt') \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right) \tag{12}$$

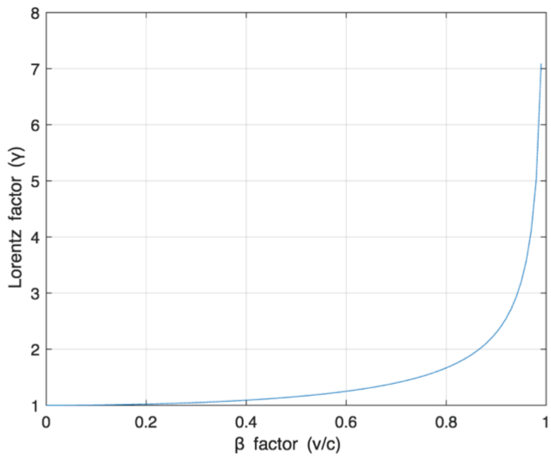


Figure 4. Graphical representation of the Lorentz factor (γ)

Note, as v<<c the fraction (v/c) approaches 0 and the denominator therefore approaches 1 which takes us back to the Galilean transformation (2), (3) and (5). Now we have illustrated that the Galilean perspective is appropriate in everyday life, but when if the speed is relativistic it will not work.

2.1.5 Derivation of Length contraction/Time dilation

The derived equations (12) give rise to both length contraction and time dilation. Imagine a rod being stationary in S'. Suppose it is traveling at speed v in the x-direction with respect to observer S. How can S determine the length of the rod? One approach is to measure the positions of its edges simultaneously and then measure its displacement $\Delta x = L$. Note, simultaneously in this case is in the S-frame. Measuring simultaneously means that $\Delta t = 0$. We modify (12) as following:

$$\Delta x' = \gamma(\Delta x - v\Delta t) \quad \Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$
$$\Delta x = \gamma(\Delta x' + v\Delta t') \quad \Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) \tag{13}$$

Since $\Delta t = 0$ we know that $\Delta x' = \gamma \Delta x$ from (13), we get:

$$\Delta x' = \gamma(\Delta x' - v\Delta t) = \{v\Delta t = 0\} = \gamma \Delta x \tag{14}$$

Note, $\Delta x' = L_0$ is the length of the rod in S' (its rest frame) and because $\gamma > 1$, the length S will observe in (14) is shorter than its length observed in S'. In (14) we can see that the only physical quantity that effects the contraction of the rod is the speed of it. It is important to note that the measurements of the edges positions are made simultaneously in the S-frame. This derivation can also be done by inserting the term $\Delta t = 0$ into the inverse transformation for time and get that $\Delta t' = -v\Delta x'/c^2$. This approach gives an illustration of time being different between reference frames. While being simultaneous in S-frame it was not in S' frame.

For time dilation suppose you place a stationary clock in S'. If a time interval $\Delta t'$ is measured in S', then what time interval would be measured in S? Because the clock was stationary in S' it implies that $\Delta x' = 0$. Inserting that into the transformation of positions in (13) gives $\Delta x = v\Delta t$. Inserting that into transformation of time in (13) we get:

$$\Delta t' = \gamma\left(\Delta t - \frac{v^2 \Delta t}{c^2}\right) = \gamma \Delta t \tag{15}$$

In (15) $\gamma > 1 \rightarrow \Delta t' > \Delta t$. The time interval measured by an observer in S will be smaller than in S'. The transformation between velocities and transformation between angles can be seen in Appendix C.

2.1.6 See & Observe

As mentioned in the introduction, we make a distinction between *see* and *observe* as follows. If I see the shape of an object that is traveling, it is referred to photons from different parts hitting the eye simultaneously in *my* frame of reference. An observation of its shape is referred to simultaneous measurements of two points of the object. That is the photons that simultaneously *left the surface* of the object. Hence the speed of light is finite, the light that hit your eye (see) was *not* the photons emitted simultaneously from the object (observation). In special relativity, there are effects where both observations what we see are the same e.g. the Doppler effect. Looking at an object you would say that it has certain colors and an observation using a spectrometer would give a value that is consistent. Studying the properties of light e.g. its color only requires an observation of one photon. When we study shapes of objects, we must determine the positions of at least two photons at the same "emission time". That aspect is not needed studying the color of the photons. However, as mentioned earlier there are effects where this is not the case and as we teach about relativistic effects we are likely to receive questions about what a relativistic environment looks like. This is where we must be aware that effects we see and observe can be different.

Example: Suppose a ruler is approaching you as mentioned in the introduction section the length of a ruler in motion can be observed by letting it pass through a detector and record when the edges pass it. Hence, we know its velocity we can calculate its length. If the ruler travel at a relativistic speed, the Lorentz transformation gives a value of the ruler being shorter compared to the rest length. However, see events means that photons hit the eye simultaneously in my frame of reference. Suppose each edge of the ruler emits one photon simultaneously in your reference frame. Since the speed of light is finite, a consequence is that the photons emitted from each edge of the ruler reached you at different times (Terrell 1959). Unless the ruler appears perpendicular to you, the photon emitted from one edge reached you before the one from the other edge.

2.2 Visual effects

This subcategory emphasizes the theory about visual effects in special relativity and how they appear in the game "A Slower speed of Light".

2.2.1 A slower speed of light

The relativistic effects cannot be experienced by students directly. But we can expose them to a relativistic environment through computer simulations. MIT game lab developed the game "A slower speed of light" which was a part of the open relativity project (Sherin 2016). The intention was that students could develop intuition about relativity if they were exposed to a relativistic environment (Kortemeyer 2013). The game is a 3-dimensional environment where you are supposed to collect orbs. Each orb successively slows down the speed of light. When all orbs are collected the max speed of the player approaches c. While playing the game they player can adjust its walking speed and the environment in the game starts to look different as orbs are collected. Starting from rest, the player can notice that the effects become stronger as the player approaches its maximum walking speed. As we have shown in equation (11) and figure 4 the Lorentz factor become greater as the fraction v/c approaches 1. You will see that objects start to change colors and get curved. Those are the effects of the Relativistic Doppler effect, Lorentz transformation etc. The game illustrates what Lampa (1924), Terrell (1959) and Penrose (1959) developed about differences between see and observation. They concluded that the length contraction can be seen perpendicular to its direction of motion. They also explained if the velocity of the object is large enough then you could see parts of a different side of the object. That means it appears to be rotated. The visualizations can be divided into two subcategories: Optical that describes shifts in colors and brightness and kinematic which describes the distortions.

2.2.2 Optical effects

This section focus on the optical effects in the game.

2.2.2.1 *Relativistic Doppler effect*

As we travel at relativistic speed there are optical effects that become visible such as the Doppler effect. Non-relativistically, the Doppler shift is describing how wave properties such as frequency is altered by motion.

$$f' = \left(\frac{1}{1+\frac{v\cos(\theta)}{c}}\right) f \quad \{f = \frac{c}{\lambda}\} \quad \lambda' = \left(1 - \frac{v\cos(\theta)}{c}\right)\lambda \tag{16}$$

In equation (16), c corresponds to the propagation speed of the wave which in this case is the speed of light. In a relativistic case the result is not the same. The beginning of the derivation is the same as for the non-relativistic case. Suppose a light source (S) is emitting a photon at an arbitrary direction θ to a detector (S') that is traveling horizontally at speed v (see figure 5). In the S frame the photon have the properties:

$$f = \frac{c}{\lambda} \quad T = \Delta t = \frac{1}{f} \tag{17}$$

Likewise, in the S' frame the photon have the properties:

(18) $$f' = \frac{c}{\lambda'} \quad T' = \Delta t' = \frac{1}{f'}$$

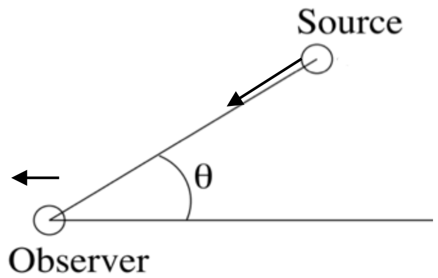


Figure 5. The light ray from a source (S) to an observer (S') at an angle θ. Credit Minahan (2011).

From the Galilean perspective, there is a universal time and therefore $\Delta t' = \Delta t$. But in a relativistic situation we must take the time dilation into account. The time between reference frames are related by:

(19) $$\Delta t' = \gamma \Delta t$$

When the photon was emitted from the source (S) it takes a certain amount of time for the photon to reach the detector (S'). During that time, the detector traveled a distance d'. That distance can be expressed as following:

(20) $$d' = vcos(\theta)\Delta t'$$

The angle in equation (20) represent where we look with respect to our direction of motion (see figure 5). The time it takes for the photon to travel distance d' can be calculated by dividing with c. Therefore, the time the detector receives the photon is the sum between the time in its reference frame and the time it took to travel distance d'. The time it is received gives information about the received frequency.

(21) $$T'_{rec} = \frac{1}{f'_{rec}} = \Delta t' + \frac{vcos(\theta)\Delta t'}{c} = \left(1 + \frac{vcos(\theta)}{c}\right)\Delta t'$$

We can transform the time between the reference frames by substituting $\Delta t'$ in (21) with equation (20). We can also use information from equation (17) in (21) and say that Δt is related to the emitted frequency of the photon by following:

(22) $$\frac{1}{f'_{rec}} = \left(1 + \frac{vcos(\theta)}{c}\right)\gamma\Delta t = \left(1 + \frac{vcos(\theta)}{c}\right)\frac{\gamma}{f_{em}}$$

In equation (22) we can see that if we multiply with the speed of light c we can express the light emitted and received in terms of its wavelength instead of frequency.

(23) $$\lambda'_{rec} = \left(1 + \frac{vcos(\theta)}{c}\right)\gamma\lambda_{em} = \frac{\lambda_{em}(1+\beta\cos(\theta))}{\sqrt{1-\beta^2}}$$

In equation (23), $\beta = v/c$ and it is the relativistic Doppler effect. We have now shown that if an object is traveling at v *away* from an observer the wavelength will increase, which means that the perceived color will be shifted more to the red spectra. If the situation would be the opposite where the detector travels *towards* the light source, we get a negative velocity $(-v)$. Then the relativistic Doppler effect looks like:

$$\lambda'_{rec} = \frac{\lambda_{em}(1-\beta\cos(\theta))}{\sqrt{1-\beta^2}} \tag{24}$$

If this is the situation the opposite happens, the wavelength appears shorter. That means the photon received would be shifted more to the blue spectra. These two phenomena are referred to as blue and redshift and they have been important to research in cosmology. It was by determining the redshift to the most distant galaxies in our universe where we could observe that it is expanding. Blue and redshift also allow us to observe light that is Ultraviolet (UV) and Infrared radiation (IR) because they are blue and red shifted. Again, the distinction between the non-relativistic and relativistic Doppler effect is that in the non-relativistic version we can assume that there is a universal time which implies that $\Delta t' = \Delta t$. In the relativistic case, we must include the time dilation. Comparing equations (24) and (16) we can see that the Lorentz factor gives a contribution to the Doppler shift. There is one from the non-relativistic result depends on the angle θ, which implies that there are cases where there the Doppler effect is pure relativistic. That case is when we look perpendicular to the direction of motion $(\theta = 90°)$. What happens is that $\cos(\theta) \to 0$ and the equation of relativistic Doppler effect then looks like:

$$\lambda'_{rec} = \frac{\lambda_{em}}{\sqrt{1-\beta^2}} = \gamma\lambda_{em} \tag{25}$$

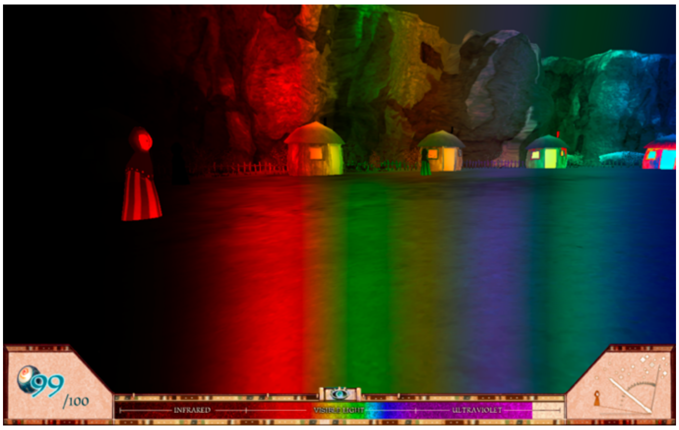


Figure 6. The observer is looking perpendicular to the direction of motion at approximately 0,9c. There is no contribution from the non-relativistic Doppler shift.

When $\theta = 90°$ we only see the relativistic contribution to the Doppler effect which comes from the time dilation. In figure 6 we have collected 99 orbs and the player has reached its maximum speed. Looking perpendicular there is green light which we will use as a reference point to calculate the velocity of the player by using different spectra. In figure 6, there is a strip where the environment appears unchanged by the Doppler effect. We use that to estimate the velocity of the player. Let ϕ be an angle that starts perpendicular to the direction of motion giving:

(26) $$\lambda'_{rec} = \frac{\lambda_{em}(1-\beta\cos(90°+\phi))}{\sqrt{1-\beta^2}}$$

In (26), we can estimate ϕ by calculating how much the unchanged spectra covers our field of view. We start from the center (eye) in figure 6 and measure the length to where the unchanged spectra begin ($d_1 = 1{,}3\ cm$) and where it stops ($d_2 = 2{,}2\ cm$). Then we divide each by the length of the field of view ($D = 7{,}2\ cm$). That quote can therefore be multiplied by 180 degrees to get ϕ. Doing that gives an interval:

(27) $$\phi \in \frac{d_i}{D} * 180° = [32{,}5° - 55°]$$

Note that ϕ above is oriented clockwise and therefore the angles are negative. Also note as we use the unchanged spectra where $\lambda_{rec} = \lambda_{em}$ implies that $\lambda_{rec}/\lambda_{em} = 1.$ Inserting the boundaries in (27) each into (26) and calculating the *median* of the velocity interval gives:

(28) $$\tilde{\beta} \approx 0{,}9 \rightarrow \tilde{v} \approx 0.9c$$

It is convenient to use the unchanged spectra to estimate the velocity as we know at that location the proper and observed wavelength are equal. Estimating the velocity using a shifted spectrum, we need to make an extra approximation by estimating the wavelengths. However, we can use the calculated $\tilde{\beta}$-factor to make other predictions e.g. where the red light is located. Starting from the center we see green light. We use the median of each spectra (green/red) giving $\lambda_G = 542{,}5\ nm$ and $\lambda_R = 682{,}5\ nm$. We use (26) and (25) to make a substitution of the proper wavelength λ_{em}

(29) $$\lambda_{em} = \frac{\lambda_G}{\gamma} \quad \lambda_{em} = \frac{\lambda_R}{\gamma(1-\beta\cos(\phi+90^0))}$$

Combining the expressions for the proper wavelengths in (29) we get that:

(30) $$\cos(\phi + 90^0) = \frac{1}{\beta}\left(1 - \frac{\lambda_R}{\lambda_G}\right) \Longrightarrow \phi \approx 16{,}47°$$

Using the same method as when we calculated $\tilde{\beta}$ in (28) we can conclude that the value of ϕ in (30) is within the red spectra in figure 6. Playing the game, we can also observe that light from invisible spectra (at rest) becomes visible. In figure 7 there is a scenario where we can observe light from both UV and IR spectra. Our speed is the same as in figure 6. We travel towards an object at $\tilde{v} \approx 0{,}9c$. At the middle of the tallest mushroom there is green light. We assume it has wavelength used in (29). We use equation (24) to calculate its actual wavelength λ_{em}. We calculate the emitted wavelength by:

$$\lambda_{em} = \frac{\lambda'_{rec}\sqrt{1-\beta^2}}{(1-\beta\cos(\theta))} = \frac{542{,}5\sqrt{1-0{,}9^2}}{0{,}9} nm \approx 2{,}5\ \mu m \in [0{,}74\ \mu m - 1000\ \mu m]$$

The emitted wavelength is blue shifted from the infrared spectra into visible light. If we do the opposite by traveling away from the same object, UV light would be redshifted into visible light (see figure 7b). We use (23):

$$\lambda_{em} = \frac{\lambda'_{rec}\sqrt{1-\beta^2}}{(1+\beta\cos(\theta))} = \frac{542{,}5\sqrt{1-0{,}9^2}}{1{,}90} nm \approx 118\, nm \in [1\, nm - \; 380\, nm]$$

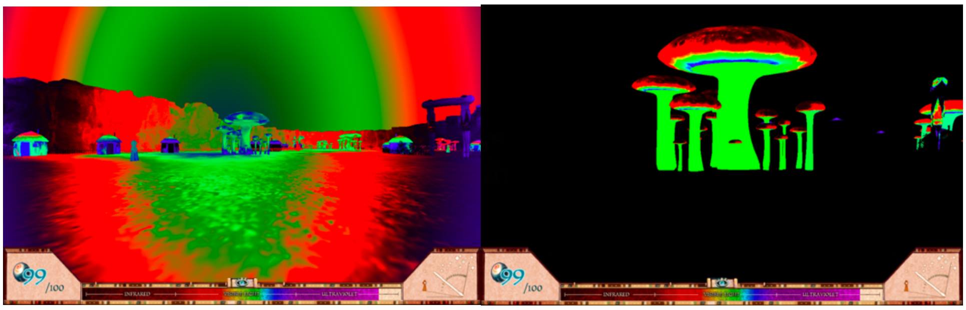


Figure 7. a) An observer travel straight ahead at 0,9c. This is the situation where the non-relativistic Doppler shift have its greatest contribution. b) same as in a) but the velocity is in the opposite direction.

In figure 6, look at the right side of the green spectra. There is yellow light which do not follow the rainbow pattern. Van Acoleyen (2018) was simulating relativistic effects and ended up in a similar situation. He explained that the reason the sky does *not* follow the rainbow pattern is because it has a mixed spectrum of wavelengths it does not consist of monochromatic light e.g. blue but of all wavelengths starting from UV to IR. What you observe is UV light being redshifted to yellow and red light. Therefore, the Doppler effect produce the pattern in figure 6. That also explains why the spectra next to the yellow appear unchanged by the Doppler effect. However, if the spectrum of the environment in the game was monochromatic e.g. yellow, then you would see a pattern that follows the rainbow and you would see yellow light at the interval in (27) to where you look perpendicular respect to the direction of motion.

Another case where unexpected colors can occur is when the angle ϕ in (26) is close to 0. That can give a situation where the non-relativistic contribution and the relativistic contribution have the same size and cancel out. In (26), we divide both sides by the proper wavelength and make a Taylor expansion of the denominator. Small angles give the following expansion:

$$(31) \qquad \frac{\lambda_{rec}}{\lambda_{em}} = 1 - \frac{v\sin(\phi)}{c} + \frac{v^2}{2c^2} \approx \{small\ angles\} = 1 - \frac{v\phi}{c} + \frac{v^2}{2c^2}$$

The negative sign of the $\phi v/c$ term gives situations where they cancel out. In classical mechanics, there is no Doppler shift if it is being observed perpendicularly. In equation (31) the trigonometric factor is zero and the observed property of the wave is same as the proper one. Unlike the geometry of objects in an environment, the relativistic Doppler effect give the same predictions as those you see. While traveling observing a green photon we would say:

"Okay in my reference frame it should have a wavelength approximately 540 nm."

Determining the wavelength using a spectrometer would give us a similar amount of nm.

2.2.2.2 *Searchlight effect*

There is one extra effect that is optic but it follows from a kinematic effect called relativistic aberration (see equation 32). It tells us as we travel towards an object its solid angle decreases and different object around us appear centralized (see figure 6). It means that our field of view will contain more photons. Which means that the intensity will increase. Intensity can be thought of as a flow of photons. High intensity means many photons that hits a surface at a time t. Therefore, a relativistic environment will look brighter. Likewise, it would look darker if we are receding an object. But remember when we see objects that is photons hitting our eye. If you are traveling too fast then the angle between you and the path of the photon cannot be too great. Traveling close to the speed of light looking behind. The only photons that will reach are those that have small angles. It is called the searchlight effect (see figure 8). It can be noticed in the game by looking perpendicular to the direction of motion in figure 6, look at the hut to right and compare it to ghost in the darkness. The red color is brighter at the hut.

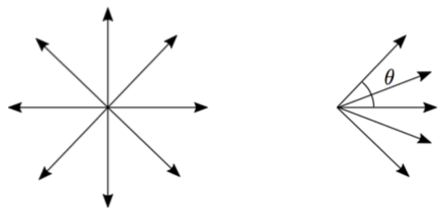


Figure 8. Illustration of how the photons partly get centralized into a cone.

2.2.3 Kinematic effects

This section focus on the kinematic effects in the game.

2.2.3.1 *Relativistic Aberration*

Since light has a finite speed, traveling at a relativistic speed, then the environment will be perceived different compared to slow speeds. If you walk straight ahead at a relativistic speed, the distance towards different objects e.g. houses, fences will not appear closer as we might expect from (14). In fact, the objects you are approaching appears more distant. We can make a comparison between how the environment looks at rest and at relativistic speed in figure 9. The explanation of why the white gate appears more distant is because of relativistic aberration. If a photon hit a stationary observer at an angle θ to the direction of motion the photon is perceived at an angle θ'. The relation between these angles is as following:

$$\tan\left(\frac{\theta'}{2}\right) = \sqrt{\frac{c-v}{c+v}}\tan\left(\frac{\theta}{2}\right) \tag{32}$$

In equation (32) c is the speed of light and v is the speed of the observer. In figure 10, look at the white gate. It appears to be more distant compared to at rest. What actually happens is that the solid angle is decreasing (Kraus 2007). Every point in the picture is partly centralized to a particular point (see figure 9) which is the one that is in the direction of motion.

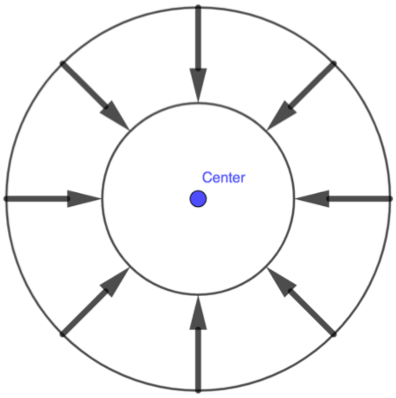


Figure 9. Illustration of relativistic aberration where all points on the outer disk (in rest frame) is centralized to the smaller disk (while moving) at speed v.

However, if the observer is traveling away from an object then the opposite happens the, the solid angle increases and it appears that you are approaching the object. Note the positions in Figure 10 a, b, c is taken at approximately the same position. Comparing 10a and b you can see objects behind you. That is because the angle between your direction of motion and the object behind you is decreasing.

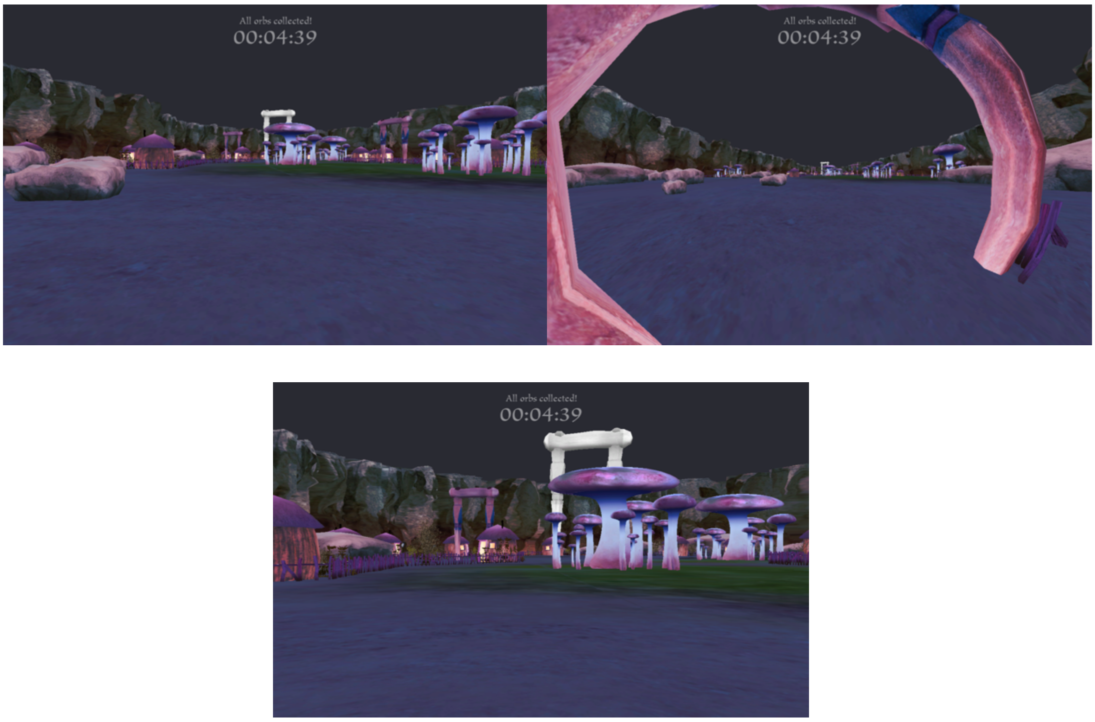


Figure 10. a) the environment at rest while in b) you are approaching the gate close to the speed of light. At c) you are moving away from it.

Approaching the white gate the distance appears to increase. The equations of length contraction (14) states that the length of a ruler will be shorter while traveling at a relativistic speed. What we saw in figure 10 was different, but that does not imply that the length contraction is unreal it only implies that looking in the direction of motion is not a good option to see the length contraction. In order to see the actual length contraction, we must look perpendicular to our direction of motion.

2.2.3.2 *Apparent length*

Now we will derive an expression for the length that we actually see L_a. We use the same example of a rod traveling in the x-direction in 2.1.5. Suppose it travels in the x-direction at a line y=h. It can also be interpreted as the distance away from the x-axis. The length of the object is L_0 in its reference frame and L in yours. In figure 11, the edges A and B have coordinates (-L/2, h) and (L/2, h). We show that the apparent length L_a of the rod is different by the one from (14) by determining if the length changes through time.

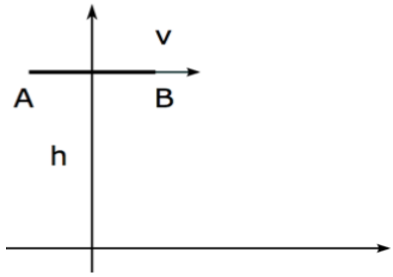


Figure 11. Setup of our system, the object travels to the right at distance h away.

The x-coordinate is determined by $x(t) = x_0 + vt$ where x_0 is the initial position of a *point* and v is the velocity of the rod. As mentioned in the derivation of length contraction, to see events mean that photons hit your eye simultaneously in *your* reference frame. Light has a finite speed which implies that before the rod passed you. The photons that hit your eye was not emitted at the same time. This is the difference to what was assumed in the derivation of length contraction (see section 2.1.5). Also, there is a difference in when the photons were emitted (t_{em}) and when they hit the eye (t_{rec}). In figure 11, the moment the photons were emitted t_{em} from A and B will not be same moment as the time they reach the eye. The time a photon reached you can be determined by following:

(33) $$t_{rec} = t_{em} + \frac{d}{u} = \{u = c, d = \sqrt{h^2 + x^2}\} = t_{em} + \frac{1}{c}\sqrt{h^2 + (x_0 + vt_{em})^2}$$

The apparent length of the rod L_a can be determined by calculating the difference in the x-coordinates $(x_B - x_A)$, we also know the coordinated of points A and B and the length L and L_0 are related by the inverse Lorentz factor $1/\gamma$:

(34) $$L_a = (x_{0B} + vt_B) - (x_{0A} + vt_A) = L + v(t_B - t_A) = \frac{L_0}{\gamma} + v(t_B - t_A)$$

We must find an expression for $(t_B - t_A)$. Note that (34) is describing what we would see and not observe as we use different emission times. In equation (34) we can notice that in order for the length contraction to match with (14), only happens when $t_B = t_A$. That is the case where an observation of length contraction match with what we see. Now let the received time $(t_{rec} = t)$ in (33) be expressed in terms of the emission times t_B and t_A giving equations:

(35) $$t = t_B + \frac{1}{c}\sqrt{h^2 + \left(\frac{L_0}{2\gamma} + vt_B\right)^2} \quad t = t_A + \frac{1}{c}\sqrt{h^2 + \left(-\frac{L_0}{2\gamma} + vt_A\right)^2}$$

Solve for t_B and t_A in (35) gives following equations:

$$(36) \qquad t_B = \gamma^2\left(t+\frac{L_0\beta}{2\gamma c}\right) - \sqrt{\left(\gamma^2\left(t+\frac{L_0\beta}{2\gamma c}\right)\right)^2 - \gamma^2\left(t^2-\frac{h^2}{c^2}-\frac{L_0^2}{4\gamma^2c^2}\right)}$$

$$(37) \qquad t_A = \gamma^2\left(t-\frac{L_0\beta}{2\gamma c}\right) - \sqrt{\left(\gamma^2\left(t-\frac{L_0\beta}{2\gamma c}\right)\right)^2 - \gamma^2\left(t^2-\frac{h^2}{c^2}-\frac{L_0^2}{4\gamma^2c^2}\right)}$$

Now we can find an expression for $(t_B - t_A)$ by subtracting (36) by (37). The apparent length L_a can then be expanded into:

$$(38) \qquad L_a = \gamma L_0 - v\left[\sqrt{\left(\gamma^2\left(t+\frac{L_0\beta}{2\gamma c}\right)\right)^2 - \gamma^2\left(t^2-\frac{h^2}{c^2}-\frac{L_0^2}{4\gamma^2c^2}\right)} - \sqrt{\left(\gamma^2\left(t-\frac{L_0\beta}{2\gamma c}\right)\right)^2 - \gamma^2\left(t^2-\frac{h^2}{c^2}-\frac{L_0^2}{4\gamma^2c^2}\right)}\right]$$

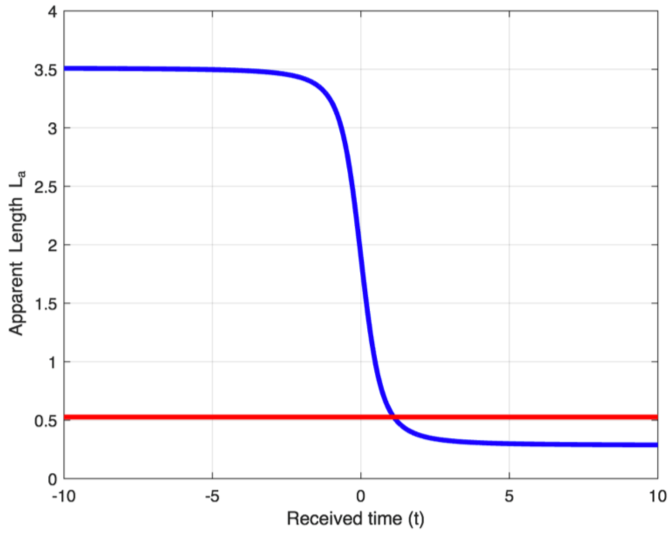


Figure 12. Graph of (38) at $\beta = 0.85 \rightarrow \gamma \sim 2$. We can see that the length of rod appears different at different times. The red line is (14).

What figure 12 illustrates is that the length you actually would see appears different at different times. It only matches with the prediction of (14) at a particular point t_L. When t < t_L (time when the photons reached the eye) the length appears longer than the prediction of (14) and when t > t_L it appears shorter. The intersection between with the line $L_a \sim L_0/2$ and the graph, is the point mentioned about (34) where the emission times $t_B = t_A$. That is also when rod appears to be perpendicular with respect to the observer. This effect can be seen in the game. In figure 13a the player is at rest and at 13b the player travels to the left and perpendicular to the gate at a speed of approximately $\tilde{v} \approx 0{,}7c$ (using same method as optical predictions). While traveling look at left side (t < t_L) of the gate and you can see that there are fewer huts compared to when at rest, in figure 13b second hut is not in the picture. The distance to them were elongated. Likewise, if you look to the right side (t > t_L) of the gate you can notice that there are more huts when at rest. If you look at the mountains you can also see that the left side appears elongated while the right side appears contracted. The lengths illustrated by the right arrows in figure 13 was determined to be contracted by 0,7. In (14) the inverse Lorentz factor gives a contraction of 0.72. Note that the player cannot collect to many orbs as the relativistic Doppler effect will shift the light into invisible spectrum (see figure 6).

Figure 13. Illustration of apparent length. The player is traveling approximately at 0,7c is in a) at rest, b) in motion to the left.

A different approach is the method that Deissler (2005) performed. He illustrated that the length contraction can be found by determining where change of the apparent length is equal to $1/\gamma$ at the origin (same prediction as mine but expressed in position rather than time). He is also including 3-dimensions. Suppose an object is traveling in the x-direction at speed v at a distance d in the y-direction and that the z-direction is pointing out of the paper. Deissler (2005) assumed that it is far away and therefore can be thought of as a point. He called it (x(t), y, z). Only x is time dependent because there is no motion in y or z-direction. Since it takes a certain time Δt for light to hit your eye which he refers to as the origin of the system, it appears to come from a different place. Deissler (2005) call that point (x_a, y, z). The y, z coordinates do not change because of the same reason as in the Lorentz transformation. The apparent coordinate x_a is related to x by following:

(39) $$x = x_a + v\Delta t$$

When Deissler (2005) say that light hit our eye that is a photon traveling at the speed of light. Therefore, the distance to the origin from the apparent point is $c\Delta t$. It can be described in terms of the spatial coordinates as following:

(40) $$c\Delta t = \sqrt{x_a^2 + y^2 + z^2}$$

By combining equations (39) and (40) we can write an expression for the proper (rest frame) spatial coordinate x as following:

(41) $$x = x_a + \beta\sqrt{x_a^2 + y^2 + z^2} \quad \{\beta = v/c\}$$

The apparent x-coordinate can also be expressed in terms the proper spatial coordinates by solving equation (41):

(42) $$x_a = \frac{x-\beta\sqrt{x^2+(1-\beta^2)(y^2+z^2)}}{1-\beta^2}$$

From the Lorentz transformation, that x and x' are related by equation (previous chapter). The remaining spatial coordinates y and z are unchanged. If two observers from different reference frames have the same origin and compare their measured lengths they would say that $x = x'/\gamma$. Inserting that into equation (42) and simplifying all β-terms give:

(43) $$x_a = \gamma\left(x' - \beta\sqrt{x'^2 + y^2 + z^2}\right)$$

Now Deissler (2005) take the derivative of (43) with respect to x'. This quantifies the change in apparent length. As the object is approaching it appears longer and when it is receding it appear shorter. The only case when apparent length of the object is the same as the length calculated by the Lorentz transformation is when its direction of motion is perpendicular to our field of view. We quantify the change in apparent length by:

(44) $$\frac{\partial x_a}{\partial x'} = \gamma\left(1 - \frac{\beta x'}{\sqrt{x'^2+y^2+z^2}}\right)$$

Evaluate (44) at $(x_0', d, 0)$ and get:

(45) $$\frac{\partial x_a}{\partial x'} = \gamma\left(1 - \frac{\beta x_0'}{\sqrt{x_0'^2+d^2}}\right)$$

Now Deissler (2005) derives that if $x_{0a} = 0$ then the object would appear shorter by a factor $1/\gamma$. If we plug in $x_{0a} = 0$ into (41) we get $x_0' = \beta\gamma d$. Inserting that into (45) we get that $\partial x_a/\partial x' = 1/\gamma$. But if $x_{0a} < 0 \rightarrow x_0' < \beta\gamma d$, which means that the apparent length of the object is greater that $1/\gamma$. Likewise, if $x_{0a} > 0 \rightarrow x_0' > \beta\gamma d$, which means the apparent length will be shorter than $1/\gamma$. In figure 14, we can see that when x'= 0 then that $\partial x_a/\partial x' = 1/\gamma$.

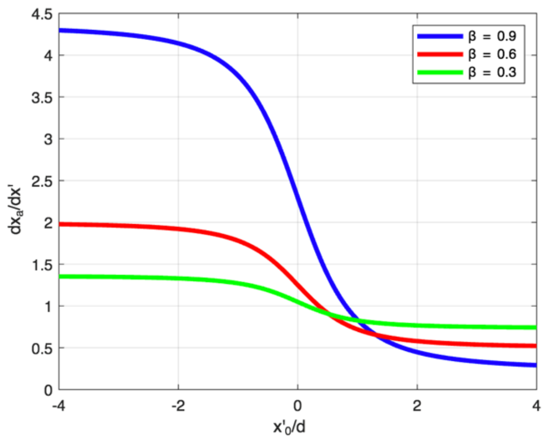


Figure 14. Graphs of equation (45): factor that describes the changed length for different β. The blue one is when $\beta = 0{,}9$, the red is $\beta = 0{,}6$ and the green is $\beta = 0{,}3$.

2.2.3.3 *Circular objects*

From (38) an interpretation is that apparent shapes of objects change. That is not always the case. There are objects where the apparent shape will be unchanged yet it would be contracted (theoretically). One of those objects is the sphere, if it moves rapidly it will maintain circular outlines (Penrose,1959). Hollenbach (1976) also showed that by proving that its diameter maintains 2r where r is the radius. He shows that the sphere can be thought of as a pile of disks of different radius. If one of those disks travel in the x-direction (from left to right) it would look like a line (the diameter) to the observer. Following assumptions were made by Hollenbach:

i) The sphere was seen when its direction of motion was perpendicular to our sight.

ii) The solid angle is small, therefore the light rays from the edges are approximately parallel.

Hollenbach (1976) treat two particular events of the disk. Event A is the light from the left edge (x_A, y_A) and event B is the light from right edge (x_B, y_B) (see figure 15). In event A, the light was emitted from a part that is behind the disk. The observer is able to see that because the disk is traveling at speed v and if it is great enough, then the disk moves out of the photons path to the observer. Projecting the emission points on the x-axis give points P and Q (see figure 15).

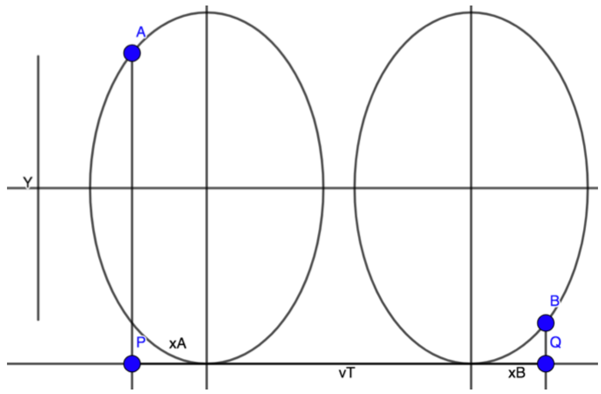


Figure 15. Illustration of Hollebachs derivation

The observer would see a line with length PQ. Because the motion of the disk is in the x-direction, that is where the contraction happens. Hollenbach (1976) first defined the contracted disk as following:

$$(\gamma x)^2 + y^2 = r^2 \tag{46}$$

If we rewrite (46) as a function y(x) and differentiate it, we get a relation of how the y coordinate change as the disk travel along the x-axis:

$$y = \sqrt{r^2 - \gamma^2 x^2} \Longrightarrow \frac{dy}{dx} = -\frac{\gamma^2 x}{\sqrt{r^2-\gamma^2 x^2}} = -\frac{\gamma^2 x}{y} \Leftrightarrow \frac{y}{x} = -\gamma^2 \frac{dx}{dy} \tag{47}$$

An important aspect is that each the two events A and B is that dy = cdt and dx = vdt. Using this aspect in (47) we can rewrite it into the following:

$$\frac{y}{x} = -\gamma^2 \beta \tag{48}$$

In equation (48) $\beta = v/c$. Substituting the x and y in equation (46) with the result of (48) we get the x and y coordinates of both emission points as follows:

$$x = \mp \frac{r}{\gamma^2} \quad y = \pm \beta r \quad \Rightarrow \quad (x_A, y_A) = \left(-\frac{r}{\gamma^2}, \beta r\right) \quad (x_B, y_B) = \left(\frac{r}{\gamma^2}, -\beta r\right) \tag{49}$$

Hollenbach (1979) also states, because the disk is traveling at speed v, when the photon reaches the observer the disk has traveled a certain distance (d'). Which can be expressed as $d' = v\tau$ (see figure 15) where τ is the time the photon traveled vertically from the point A to B. The distance PQ can then be expressed as following:

$$PQ = -x_A + v\tau + x_B = \frac{r}{\gamma^2} + v\tau + \frac{r}{\gamma^2} = \frac{2r}{\gamma^2} + v\tau \tag{50}$$

The term $v\tau$ in (50) can be expressed in terms of the y coordinates. First the time τ can be expressed as the total vertical displacement from (49) divided by the speed of light. By multiplying τ with speed v we get following:

$$\tau = \frac{Y}{c} = \frac{|y_B - y_A|}{c} = \frac{2\beta r}{c} \Longrightarrow v\tau = \frac{2\beta v r}{c} = 2r\beta^2 \tag{51}$$

Insert (51) into (50) and get:

$$PQ = \frac{2r}{\gamma^2} + 2r\beta^2 = 2r\left(\beta^2 + \frac{1}{\gamma^2}\right) = 2r\left(\frac{1+\beta^2\gamma^2}{\gamma^2}\right) = \left\{\gamma^2 = \frac{1}{1-\beta^2}\right\} = 2r \tag{52}$$

From the result in (52), we have derived that the line will appear to have a length of 2r which match the diameter of the disk as Hollenbach (1979) predicted. Hollenbach (1979) also states that because of the assumption of a sphere consisting of a pile of disks and that we are working with small solid angles the sphere will not appear contracted. Because the observer sees all points from event A to B, where A was behind the disk (see point A in figure 15). Therefore, it appears that the disk has been rotated by an angle θ counterclockwise. That angle is related to coordinate from event A $(y_A = \beta r)$. That coordinate is the opposite in a right-angled triangle and the radius r being the hypotenuse. Therefore, the angle that the disk appears to have rotated is:

$$\sin(\theta) = \frac{\beta r}{r} = \beta \tag{53}$$

If the solid angle would be larger, then the apparent shape of the sphere would be the same (Boas 1961) but the lines would appear different (Müller & Boblest, 2014). The lines of the of the sphere appear curved. The rotation that (53) describe can be seen in Figure 16 where the textures appear rotated counter clockwise. It can also be seen that the diameter appears unchanged. The poles can be thought of as a pile of disks that have the same radius. We also see that the poles appear curved because it takes light longer time to reach our eye and therefore the bottom appears to pass us before the top. One conclusion we can draw from Hollenbach (1979) is that the earth will *not* appear flat.

Figure 16. The textures at the poles appear rotated traveling left. And the poles appear curved because it takes longer time for the light to reach the eye from the top.

2.3 Educational

This subcategory emphasizes the theory about educational science.

2.3.1 Visualizations of concepts in physics

There is research which supports that an interactive computer simulation can meet the needs required for effective learning. Bransford (2000) reported that the actual learning of the students improves as they create their own understanding of scientific ideas from prior knowledge. That was important to Wieman et al (2008) who meant that in order to accomplish that students develop their understanding from prior knowledge, they need to be motivated to engage with the content and learn from it, which an interactive computer simulation can do. One of the biggest projects that is directly related to visualizations and simulations is the Physics Education Technology (PhET) project. Today, they have developed over 80 simulations which covers multiple topics in physics including those of a modern kind e.g. quantum mechanics. It is common that PhET simulations emphasize few concepts. Wieman et al (2008) further reports that interacting with simulations helps the user to develop their mental models, which has been tested in physics teaching. McKagan et al (2008) reported after letting students interact with a PhET simulation about the photoelectric effect, the outcome was that 80% of the group that used a simulation understood the concepts while only 20% understood it by using traditional teaching. Furthermore, Wieman et al (2008) studied how the simulations of PhET at the end leads to learning by interviewing users (N>250). In their analysis, they found that the students thought the PhET simulations were fun and engaging which is important for motivation. They also reported multiple characteristics of how the simulations were engaging. Some of those were:

1) The user could control the environment.
2) It was neither too hard or easy to face the challenges.

Even if the simulations of PhET have been effective in teaching it requires that the user interact with actively and not just watch it (Wieman et al 2008).

There are also studies of where simulations that was not developed by PhET have been tested in teaching. One example of that is Yu (2017) who was testing what impacts planetary motion simulations have. Students that took the same course were randomly selected into three groups, where each one got exposed to different teaching. The first group did not use any simulations at all. The second and third group were exposed to visualizations in their instruction but of different kinds, group 2 only saw visualizations projected on flat screen in a lecture hall while group 3 saw them in a different environment. Group 3 were taught in a digital planetarium at a museum. However, even if the groups had different teaching, all of them used the same textbooks and same content in the lectures. The impact of the visualizations was tested by weekly assessments that consisted of multiple choice questions. The outcome of that was that (findings) group 3 showed the greatest gains. Yu (2017) also claimed that the students of group 3 developed a greater understanding about certain concepts in astronomy. And therefore, her conclusion was that students who were exposed to visualizations showed better gains. She also claimed that both simulations and visualizations are powerful tools that can assist students visualizing effects that human senses cannot.

The conclusion that Yu made is not unique. Similar results have been observed in topics that are more abstract than astronomy, such as quantum mechanics. Kohnle (2015) was testing how simulations about two state systems can assist students understanding of concepts in quantum mechanics. Students performed simulations in a software called *"Phase Shifter in a Mach-Zehnder Interferometer".* The students were simulating the concept superposition by sending single photons through an interferometer. It also allowed them to vary the phase shift by inserting a phase shifter. The purpose of the simulation was to help students create a mental model of single photon interference. The gains of the simulation were measured by a pretest that the students took before they did the simulation which after the laboratory was repeated. The outcomes of Kohnles (2015) study were that they performed better after the simulation. And therefore, Kohnles (2015) conclusion of the experiment was that simulations can help students gather conceptual understanding in quantum mechanics. She also described that simulations have high potential for learning because they allow us to see invisible effects. Except the topics that the visualizations illustrate, the conclusions Kohnle and Yu make is similar. Visualizations seem to have positive outcome when it comes to what students are learning. Not only students that have high prior knowledge but to students that have low prior knowledge. Casperson (2006) instructed students to do simulations that emphasized concepts of electrostatics. In that study, it could be seen that students with low prior knowledge gain more than students with high prior knowledge by interacting with simulations. Casperson (2006) also states that visualizations have positive impacts at students understanding of concepts. Visualizations appear to be effective if they are used properly.

2.3.2 Visualization of concepts in special relativity

As for computer simulations, published research that covers visualizations of concepts from special relativity are limited and PhET has not developed a simulation about it yet. It is common to use a course about special relativity as an introductory to modern physics. McGrath (2010) states that the purpose of courses of an abstract kind not only about learning about the particular topic but to develop the students abstract thinking. But McGrath also states as students get exposed to the abstract courses they tend to focus on manipulating different equations instead of learning the concepts. He reports that might help them to get good grades at exams (quantitative). But as students' go deeper in abstract physics, their foundation is unstable. One of few studies about visualization of relativistic concepts was reported by Savage (2007). It was the Australian National University (ANU) who constructed a 3-hour long laboratory session as a part of course for first-year students. The course had nine lectures about special relativity. At the laboratory, the students were supposed to make measurements and compare it to theory. One of the instructions was to measure length contraction as a function of speed looking perpendicular to the direction of motion at a large distance. The laboratory did not use "A slower speed of light" to simulate a relativistic environment, they used "Real time Relativity". Before the laboratory, the students had preparations tasks that contained both conceptual and quantitative questions. The instructors of the laboratory measured the effectiveness by analyzing how the students interacted with "Real time Relativity", log books, surveys before and after the session. The outcome of the study was that students believed that they learned more about relativity. It was reported that some students thought it was concrete and gave them an opportunity to see the effects but there were also students that perceived it as forcing the equations into the simulation. However, Savage (2007) could also see that the students managed to make accurate measurements about time dilation, length contraction and Doppler effect. He concluded that "Real time Relativity" gave students opportunity to both discover and confront misconceptions in relativity.

The work of Savage (2007) was further used by McGrath (2010). He expanded it by including more institutions. One of them was the University of Queensland (UQ). This study was done for 2 years and over 300 students participated. The implementation of "Real time relativity" were similar to Savage (2007). The students were instructed to test it and link their observations to concepts of special relativity. In McGraths study, the tasks appear more open in comparison to the study of Savage. For length contraction, instead of restricting the exploration to look perpendicular to direction of motion, the task was to determine how length contraction appear in "Real time Relativity". This implied that students also had to distinguish between length contraction and contributions of other relativistic effects. The students were supposed to explain their observations with both concepts and drawings from special relativity. Another task was to determine what a ship that passes at a particular relativistic speed visually looks like. In the study of McGrath, the data of interest was the change of attitudes about special relativity and the learning outcomes. They collected data about student attitudes by letting the students fill a survey after their laboratory session. In the analysis, it could be seen that the students' interest in special relativity increased. The students found it more efficient and stimulating in comparison to experiments done in regular laboratories. The students were positive to use simulations for future studies. It could also be seen that students who used Real time relativity found special relativity to be less abstract than students who did not use it. Before the students used Real time relativity they took a conceptual test which they wrote two times. Before and after they used Real time relativity and there were noticeable effects in their actual learning. The students who participated showed improvements understanding the concepts in special relativity compared to before testing real time relativity. They showed improvements understanding concepts such as time dilation and simultaneity, in their exam there were improvements both on questions regarding special relativity but also on questions that are related to quantum mechanics. In the survey, the students reported reflections that are important to consider as a teacher. The reflections reported in McGrath (2010) were:

1) *"They gained an ability to visualize relativistic effects which make it easy to apply theory"*

2) *" That it was much easier to learn the concepts (ALL concepts) of relativity when it is seen visually,"*

3) *"It helped a lot with understanding because you could visualize something, that you have no experience of visualizing in real life."*

McGraths conclusion was that visualization can be a powerful tool and that it has potential of developing students abstract thinking.

Except for computer simulations, there is research which covers visual effects and teaching from a different perspective than this project. Hewson (1982) is discussing that a challenge in learning special relativity students and teachers think about objects as having an independent reality where objects have fixed properties such as mass and lengths. In that study computer simulation was not implemented but interview. While interviewing a graduate student Hewson (1982) noticed that the graduate student stated that an object only *appears* contracted because it is moving while it is constant being observed from an observer traveling along with the object. The student describes an independent reality which is a misconception in special relativity.

2.3.3 Comparison to this project

The presented findings that Wieman et al (2008) reports are interesting because they give evidence that computer simulations can engage students with the content and learn from it. Also, the first characteristic of what makes a simulation engaging, matches with how the user uses “A slower speed of light”. The user controls the direction of motion and what speed it is traveling. Therefore, the user controls the environment. The second characteristic that they reported remains to be tested. The studies of Kohnle (2015), Casperson (2006) and Yu (2017) are also of interest because they give evidence that visualization of different concepts in physics (including modern physics) can have positive impacts the students learning of concepts from different topics in physics. Their findings do not imply that visualization of concepts in special relativity will have the same outcome. But the studies of both Savage (2007) and McGrath (2010) give evidence that it is possible special relativity too.

But there are differences between this project and the studies above. PhET simulations simulate few concepts which was also the case of Kohnle (2015), McKagan (2008) and Casperson (2006). While playing “A slower speed of light” there are many effects that are simulated at the same time e.g. relativistic Doppler effect, searchlight effect, relativistic aberration and length contraction. I also use a different game engine than Savage (2007) and McGrath (2010). They used “Real time Relativity” which allows you to travel close to the actual speed of light ($c \approx 3 * 10^8 \ m/s$). That itself is not a problem because it is only a different scale compared to the one in “A slower speed of light”. As long as the user travels at the same v/c, the effects will appear the same in both simulations regardless if $c = 5\, m/s$ or $c = 3 * 10^8 \ m/s$. However, studying how a relativistic environment with Earthlike conditions traveling at the actual speed of light is difficult. It requires the user to analyze small time differences. It would be more difficult to experience what our environment on *Earth* would look like. Savage (2007) actually said that they used astronomical scales to be more realistic. In this study, the students do not use the simulation in a laboratory session but as a part of a hand in problem that contains different kinds of tasks. One that is conceptual and one that is quantitative. In the reported studies the students were instructed to analyze specific effects. In this study were free to select an effect to discuss. When it comes to the number of participants there is also a difference between this project and the studies of Savage and McGrath, they also had more preparation.

Chapter 3

3 Empirical study

3.1 Method

Two of the research questions of this project were: What effects do students report that they have noticed while playing the game and how they relate the observed visual effects and values of physics quantities arrived by applying Lorentz transformation. Therefore, I look for data about what they think that they see when they play the game. The second research question can be answered by analyzing how the students motivate their observations. What makes them think they saw effect X. This data can be collected by letting students play the game and instruct them to explain what they notice and how they notice it. One way of doing that is inviting students to a workshop where they discuss what they see in the game. But the project was limited to a short time and workshops takes lots of time collecting data. An outcome of that would be that the validity of the first research question decreases. Therefore, I decided to include the game as a part of a hand in problem which tested different abilities (conceptual, quantitative, reflection) in special relativity. The third research question can be answered by analyzing if there are any ways of reasoning that are repetitive.

The data that was collected were solutions to a hand in problem that consisted of three problems about relativity. The students had two submissions, first they submitted an individual solution. The second submission was after a 2-hour long seminar where they discussed the same hand-in problem in small groups. During that discussion, they wrote a group solution which I also collected. What I seek in the solutions are which effects the students report they see and motivations of their statements. In order to increase the validity in my findings, the game must be tested by a large number of students which implies many solutions, which is useful especially for the first and third research question. The findings are presented both as extracts from the students' and group solutions and in tables.

3.2 Hand-in problem

The hand in problem that the students solved was a part of a course called Mechanics III (1FA103). Mechanics III is a course that gives an introduction to analytical mechanics where concepts like the Lagrangian and principle of least action are the focus of instruction. The course also introduces concepts in special relativity such as 4-vectors, Lorentz transformation and tools such as tensors. As the students submitted their solutions and participated in a group seminar the main instructor of the course, provided them with an extra credit on their exam. The structure of the hand in problem was based on a different one that was used in a different course (see Appendix A). The problem had 3 tasks (see Appendix B), all of them were covering the same phenomena but from different perspectives. In the first task the students were supposed to play the game and identify at least two relativistic effects and describe the physics behind them. The second task of the hand-in problem consisted of two parts. The first part was of a quantitative kind, it described a situation where an active galaxy nucleus is emitting two jets and the task was to determine the velocity and direction of the jets relative to

Earth. To solve the problem derivations/calculations was required. The difficulty of the problem was similar to problems they solve at exercise sessions. The second part emphasizes reflection on both concepts from the first task, but also on their results from their calculations. The task was: When you look at the Andromeda galaxy through a telescope, can you see any relativistic effects?

3.3 Students

There was a mixture between the students that took this course because it is given to more than one program. Some of the students are bachelor students of physics and some of them are engineers. However, in their syllabus the students have taken similar courses in physics and mostly in mathematics too. The bachelors have taken one course in physics that the engineers have not which was thermodynamics. Except physics courses from high school, they have no previous experience of special relativity at courses taken at university (based on their syllabus). Before the hand-in problem was assigned, they only had one lecture about special relativity that covers the Lorentz transformation, length contraction and time dilation (not the visual effects).

3.4 Implementation

Before the hand in problem was published and available to the students, I played the game and solved the hand in problem in order to make a comparison between mine and the solutions of the students. The students had 1 week to solve the hand in problem. In the first round of submission, the students were allowed to collaborate but instructed to submit an individual solution. They were also instructed that it was acceptable to submit something that was wrong as long as they had made a proper attempt at answering the task. Therefore, students were not expected to score 100 % in their individual submission. The purpose was for them to get experience from the game which would be discussed at the group seminar. When they arrived at the seminar the students were instructed to divide themselves into groups of 3-4 people, not with people they know. The seminar had 18 groups and lasted for two hours. At the seminar, they wrote a group solution at whiteboards which were provided to the groups. The groups could use them to do sketches or write their entire solution and they could be a part of the submitted group solution. Because the groups were not recorded, it is possible that they have discussed the game more than it appears in their solution. Even though the groups were instructed to include that. At the seminar, there was no restriction of how they were going to distribute the time. The students were free to discuss task 1 more than task 2 as long as they presented an answer to all of the problems. If assistance was needed, the students could ask several instructors. The students did not know until the seminar that I was going to attend and listen to their discussions. While I was listening to the discussion of 2 groups. I kept a distance away trying not to interfere in their discussion to ensure that their notes are based on their own discussion. After the seminar, the groups submitted their solution and I started to analyze their solutions, both group and individual. The analysis focuses on both the group solution and the individual submission.

3.5 Ethics

The findings section present extracts from their solutions. The students were promised to be anonymous.

Chapter 4

4 Findings

This section describes what could be seen in the solutions, both the group and individual solutions. There were 56 students that handed in a solution and 18 groups that handed in a collective solution. I will refer to their solutions as following:

$$S^{\mu} = \{S^1, S^2, S^3, \dots, S^{56}\} \quad G^{\nu} = \{G^1, G^2, G^3, \dots, G^{18}\}$$

Each element in the vectors S^{μ} and G^{ν} corresponds to one student or group. The indices only illustrate that the student or group I cite is different. In task 1, I do a comparison of the differences between my solution and the submitted ones. Note that my answers also consist of the equations, reasoning and figures derived in *earlier sections*.

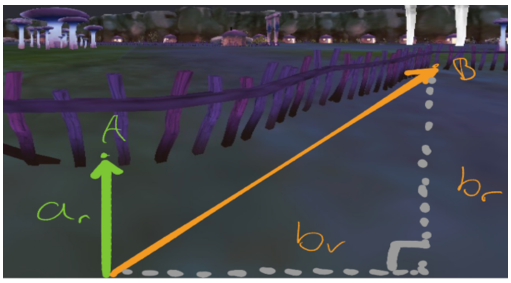
4.1 Research Question 1

The first task of the hand in problem is of relevance because that is the only task where the students were directly instructed to use the game. The effects that were frequently reported and described by the students or groups were: length contraction, Lorentz transformation, Doppler effect and searchlight effect. The effects will be divided into two subcategories optical and kinematic effects and they will be presented in tables where a comparison is made between my own answers and selected examples of students' individual and group submissions. I selected these examples because they illustrate different ways of how the students related the visual effects to physical quantities arrived at by applying Lorentz transformations, that is also relevant for research question 2. All illustrative examples are grouped by topic based on what the student reported to have noticed. That means if the student for instance reported it noticed effect A but describes it as effect B it was grouped by effect A.

Relativistic effect	Amount (N)	Groups (N)
Searchlight effect	19	6
Length contraction	18	7
Lorentz transformation	16	3
Relativistic Doppler effect	52	16
Total:	105	32

Table 1. Description of how many times each effect was reported by the students.

In table 1, we notice that the relativistic Doppler effect was the dominant one at both submissions. Note that there were 56 individual submissions and 18 group submissions. The relativistic Doppler effect has therefore been reported by almost everyone. Table 2 and 3 presents how the students described the visual effects in the game.

Kinematic effects	
Students:	**Group:**
Length Contraction S[1]: Length contraction is basically a contraction of measured distances in high velocities. The absolute length gets shorter … because $\sqrt{1 - v^2/c^2}$ will be <1, regardless if it is positive or negative, traveling towards or away from an object then distances will be shorter. S[6]: We can see that length contraction occurs. It occurs because c is constant. To maintain that, the distance must change. S[8]: Spacetime has to change because c is constant. An observer that travels fast will travel a shorter distance because of length contraction. The distance that the observer measures is $L = L_0\sqrt{1 - v^2/c^2}$ S[9]: As you go through the game you notice that the geometry of the world around you are distorted. This is an effect of the fact that the relative time difference for the light to travel to our eyes from different parts of the objects has changed, as their length decreases in some dimensions, but not others. The effect of the object decreasing in length in the dimension that corresponds to the relative velocity it has towards an observer is called length contraction. S[17]: As you travel straight ahead and backwards, the world gets zoomed out and zoomed in. What causes zooming in/out is length contraction. S[18]: Length contraction: As we move forward we experience that the distance to an object increases. **Lorentz transformation** S[27]: Figure 2 shows the Lorentz transformation which shows that the speed of light is constant, therefore spacetime has to bend.	**Length Contraction** G[2]: Our perception of space is distorted at high speeds. As we are passing an object, then our components of the velocity are distributed differently because we are passing a 3D-object. A horizontal pole appears shorter. A vertical appears bended. G[8]: At high speeds, we notice that the light travel different lengths from different parts of the object. The difference in height that the light travels make the object appear curved. G[12]: The vector a_r (figure 17) has one component only in the direction of motion, it implies that the component is exposed to length contraction. For the vector b = (b_r + b_v), the distance is longer, and it takes longer time for the light to travel to the observer. That is a result of the speed of light being finite. Since the light takes time to travel, point A will not exhibit contemporary events such as point B, and the perceived distance will be much shorter in the middle of the image, causing the room to bend backwards. Actually, the fence at point B is much closer than it looks, but since there is a time difference in the paths traveled by the light, they will be perceived as being further away. This effect is called relativistic aberration.  Figure 17. Group observation of length contraction G[5]: $x' = \gamma(x - vt)$. If we travel away from the object $v > 0$ then $x' < x$, the distance to the object will be shorter. Which was seen when the environment appeared zoomed in. If we traveled

S[42]: As you approach the speed of light the room curves.

S[16]: Looking at an object while walking sideways it appeared to bend. Not entirely sure why but I assume it is because the light from one point of an object is faster than a different point beside it.

S[26]: This can be seen at the end of the game when you travel close to c. You can see that objects around you start to curve appear to be more compact. Because $\sqrt{1 - v^2/c^2} < 1$ implies that L´ <L, the length that the observer sees is shorter than the true one. Hence, the surroundings appear to be curved and shrunk when moving close to c in the game.

S[24]: When all shells were picked and the player starts running around the objects start to curve around the player, and the players speed vary. That is because the speed of light is always in the same speed.

towards the object $v < 0$, then $x' > x$. Which was seen when the environment was zoomed out

Lorentz transformation

G[3]: The fact that light from objects further away takes longer to reach the observer. The light has different long path from the observer to the pillar's foot and top (figure 18). Therefore, the light from A will reach O at the same time as the light from B which originated at a later position. Therefore, the top appears to be to the left of the foot.

Figure 18. Student observation about Lorentz transformation

G[13]: Spacetime is not the same at low speeds as in speeds close to c. Spacetime is bent in order for postulate 2 of special relativity to match.

Me:

Length Contraction:

In the game, the length contraction does not always appear as an actual contraction. The length contraction can be seen but under certain circumstances. We derived using equation (38) that if we look perpendicular to our direction of motion the length between two poles does appear contracted as the Lorentz transformation predicts (when the emission times $t_B = t_A$). In figure 13 we can see that as we are approaching an object it appears longer and not contracted. Look at the mountains at the left side of the gate, they appear elongated. We can also see that as we are receding the mountains they look contracted that can also be seen in figure 13, we can also see a third house. The length contraction is hard to see by looking in the direction of motion. If we are traveling towards an object the distance to it does not appear contracted. It appears elongated but what actually happens is that the solid angle of the object is decreasing because of the relativistic aberration (see equation 32 and figure 10). When we say the distance appears longer that is the solid angle of the object that decreased. In classical mechanics, when we are approaching an object, the solid angle increases but in relativistic speeds it is a different situation (see Appendix C for derivation). We can also look at circular objects that have textures e.g. the poles, appears to have undergone a rotation counterclockwise (see equation 53). In figure 15, at point A is blocked by the circle at rest. However, if it is traveling at a relativistic speed the boundary moves out of the path between the eye and point A and the photon reach the eye and the object appears to have undergone a rotation. Figure 16 shows an example from the game.

Distortions:

My description of the visualized Lorentz transformation is related to my description of the length contraction. Furthermore, looking at different parts of an object in the environment, it takes light different times to reach your eye. An outcome of that is that you see changes at closer parts before

the distant parts. The game developers call that the “runtime” effect. It is related to the fact that the speed of light is finite, light emitted simultaneously will reach the eye at different times. As an observer look at objects e.g. the poles in figure 16, they appear curved. That is because as you travel to the left the light from bottom has reached you before light from the top. It therefore appears that the bottom passes before the top of the gate.

Table 2. Comparison between observations of kinematic effects.

<table>
<tr><th colspan="2">Optical effects</th></tr>
<tr><th>Students:</th><th>Group:</th></tr>
<tr><td>

Searchlight effect:
S[3]: The searchlight effect gives rise to stronger intensity of objects approaching from the side relative to the observer. It arises when light can be seen as particles that with higher density become more intense and thus stronger (brighter) and PSs darker as you move away.

S[4]: When you run in the rain you get more water drops than if you were to walk in the same way, if you move towards the light you are hit by several photons.

Relativistic Doppler effect:
S[45]: In this picture, we see that the figure turns red as it moves towards one while at the same time moving towards the figure, which is the relativistic Doppler effect and redshift.

S[46]: We can clearly see the effect of relativistic Doppler shift in Figure 1, where everything that is in the direction of motion, is shifted in wavelength towards the shorter wavelengths (green, blue, violet etc.), while Figure 2 shows that what is behind the player at a sufficiently high speed is shifted to the infrared spectrum where in the end we can see no more of the light. This is partly because the player moves in the room where the wave is propagating, and when you encounter a wave traveling towards the player's direction of travel, the waves' tops and valleys will meet the player more often, which corresponds to a higher frequency and thus a lower wavelength.

S[2]: The speed of light is constant, so if the source emits light it will propagate with c. In the next time step the source has moved, this light emitted will center around the new position and the distance between the light and the new position and the distance between light from the different positions will have decreased in the direction of source speed and increased opposite to that. The wavelength of light shrink’s vs gets longer.

</td><td>

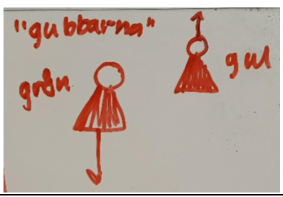
Searchlight:
G[4]: Where the player looks, the picture appears brighter. This is enhanced when the player is moving in a certain direction, as the image becomes much brighter. The reason for this is that an object moving is hit by more photons than an object at rest.

G[6]: If you travel close to the speed of light, your front will be brighter than from behind. One reason for that is because you get hit by more photons. (derivation below)

Relativistic Doppler effect:
G[9]: In the game, the observed wavelength is shifted. If we move fast towards the source the wavelength appears contracted and become shorter and the opposite for the other direction.

G[18]: If the source is traveling away from the observer in relative speed → lower frequency →longer wavelength → experience redshift that approaches the infrared spectrum

G[2]: As we move forward, we see blue light because the wavelengths are small. As we move backwards we see red light because the wavelengths become longer (figure 19).



</td></tr>
</table>

<table>
<tr><td>

- Towards you → shorter wavelengths→ blue shift
- Away from you → longer wavelengths → redshift

</td><td>Figure. 19 Group that related relativistic Doppler effect to ghosts in the game.</td></tr>
<tr><td colspan="2">

Me:
Searchlight effect:
In the game, we are able to see two optical effects. The searchlight effect and the relativistic Doppler effect. The searchlight effect is caused by the relativistic aberration as described in table 2. The relativistic aberration could be seen in figure 10 where the solid angle decreased. At that point of the game the optical effects were turned off. What happens is that all photons that hit you are concentrated to a smaller spatial angle. A consequence is that the environment appears brighter (see figure 7 and 8).

Relativistic Doppler effect:
The changes in colors comes from the relativistic Doppler effect. As we travel at relativistic speeds objects get blue shifted approaching them and redshifted leaving them. This phenomenon allows us to see light from different spectra than visible light (see figure 7). In equation (24) we showed that the relativistic Doppler effect is different from the non-relativistic one. In this case we must take (15) into account. There is a time dilation between the reference frames. In equation (25) we could also see that even if we look perpendicular to the direction of motion we see a Doppler effect which we would not in the non-relativistic case (see figure 6). In the game, it does not appear as a rainbow which is because of mixed of spectra of wavelengths and not monochromatic light (see figure 6 and the calculations). There is also a spectrum that appears unchanged which can be used to estimate my velocity in the game.

</td></tr>
</table>

Table 3. Comparison between observations of optical effects.

4.2 Research Question 2

The students were able relate quantities arrived at by applying the Lorentz transformation for some of the relativistic effects. In table 3, we can see that they were able to relate the changes in colors to the blue shift and red shift depending on the direction of motion and the description does not change during their group discussion. From table 3, we can see that they are similar:

> S^{45}*: In this picture, we see that the figure turns red as it moves towards one while at the same time moving towards the figure, which is the relativistic Doppler effect and redshift.*
>
> G^{2}*: As we move forward, we see blue light because the wavelengths are small. As we move backwards we see red light because the wavelengths become longer (see figure 19).*

This student and group linked the color shifts from the game to relativistic Doppler effect. However, the situation that they described is when the contribution of the non-relativistic Doppler effect is largest. The students thought they described the relativistic Doppler effect but they did not clearly express that it has two contributions. In task 1, they did not describe the relativistic contribution of the Doppler effect which is greatest when we look perpendicular to the direction of motion. We cannot see that they studied that situation and compared it to the non-relativistic Doppler effect. It was not until task 2a where we could see that some groups tested what happens to Doppler effect if the angle changes but their conclusion was not that as we look perpendicular to the direction of motion, then the relativistic contribution is greatest and it is different from classical mechanics. Some groups only calculated the observed wavelengths of the emitted jets and concluded that they are the same. These are cases where some groups discussed what happens if the angle changes:

> *G[4]: $\theta \rightarrow 90°$, the jets have the same shift.*
>
> *G[10]: When θ approaches $90°$, the wavelength $\lambda' \rightarrow \lambda''$ and be equal to $\lambda_0\gamma$*
>
> *G[16]: If the angle increases and approaches $90°$ the shift will decrease and get close to zero. Then, the measured wavelength will be the same as the original one.*

They present the mathematical interpretation of the relativistic contribution but they did not present any situation where they saw this or that this is the case where we only see the relativistic contribution. One effect where the majority of the students struggle relating visual effects to quantities arrived at by applying the Lorentz transformation was the length contraction. As they describe it they try to force the theory onto what they see. They reported that they saw the length contraction but they cannot explain it in a convincing way.

> *S[6]: We can see that length contraction occurs. It occurs because c is constant. To maintain that, the distance must change.*
>
> *S[8]: Spacetime has to change because c is constant. An observer that travels fast will travel a shorter distance because of length contraction. The distance that the observer measures is*
> $L = L_0\sqrt{1 - v^2/c^2}$
>
> *S[17]: As you travel straight ahead and backwards, the world gets zoomed out and zoomed in. What causes zooming in/out is length contraction.*
>
> S[18]<u>: Length contraction:</u> *As we move forward we experience that the distance to an object increases.*

None of these solutions describe a situation where they see an actual contraction or how their explanations are related to it. Yet that is what they report that they see. S[18] says that the distance increases and S[17] says that when we travel straight ahead the world gets zoomed out which is the opposite of a contraction. S[6] says it occurs because c is constant and to maintain that distance must change. By looking at their motivations of what they thought were the length contraction, was through changes in distances. From their perspective, the only effect that is related to that from the course is length contraction therefore it is likely that they thought change in lengths is the length contraction. Therefore, I speculate that the students expected the predictions of the Lorentz transformation to match with the visual effects. The majority of the explanations were in similar form but there were cases where students have presented an explanation that are more convincing by including aspects as light travel different distances from different parts of an object which gives rise to a time difference of when the light hits the eye.

S^{9}: As you go through the game you notice that the geometry of the world around you are distorted. This is an effect of the fact that the relative time difference for the light to travel to our eyes from different parts of the objects has changed, as their length decreases in some dimensions, but not others. The effect of the object decreasing in length in the dimension that corresponds to the relative velocity it has towards an observer is called length contraction.

In the group solutions, not as many try to force the theory onto what they see. They present explanations of why the environment appears as it does. This is an example of a group that reported why the length contraction does not appear as a contraction:

G^{12}: The vector a_r (figure 17) has one component only in the direction of motion, it implies that the component is exposed to length contraction. For the vector b = (b_r + b_v), the distance is longer, and it takes longer time for the light to travel to the observer. That is a result of the speed of light being finite. Since the light takes time to travel, point A will not exhibit contemporary events such as point B, and the perceived distance will be much shorter in the middle of the image, causing the room to bend backwards. Actually, the fence at point B is much closer than it looks, but since there is a time difference in the paths traveled by the light, they will be perceived as being further away. This effect is called relativistic aberration.

There were groups that tried to describe the relativistic aberration but it was not linked to the situation in figure 10. It was not used to explain why objects appear more distant but to the explain the searchlight effect.

G^{6}: If you travel close to the speed of light, the front will be brighter than the back. One reason is because you get hit by more photons. (derivation below)

The derivation G^{6} mention is for relativistic aberration which match with mine (see Appendix C). Because of the mixture of answers, it is likely that each group just selected the solution that where most convincing. In the group submissions, it was common that the groups used a document that was submitted individually. But that also implies that they during their discussion managed to distinguish between correct and incorrect interpretations of what they saw in the game.

4.3 Research Question 3

Playing the game, it is hard to study one effect at the time. G^{12} tried to describe how they saw length contraction but could not separate it from the effects of relativistic aberration. When it comes to the optical effects it was easier for the students to explain what they see. Therefore, the game could be productive when students are learning about the relativistic Doppler effect. As for length contraction, there are indications that the students realized that the prediction of the Lorentz transformation did not match with what they reported. When they said they saw the length contraction they did not describe any particular situation of where they saw it. If they did not realize that they are different, then the individual submissions would have contained more screenshots of random situations where they say that this is the length contraction. We see that students tend to only look in the direction of motion. Studying optical effects, that is not problematic but it is harder to study the length contraction by looking in the direction of motion. Therefore, to use the game productively the students need to be encouraged to look in different directions without revealing the outcomes. Note that the students only had 1 lecture about special relativity as preparation from the course that did not cover visual effects. It indicates that the students also could use an introduction to visual effects.

One way of using this in teaching was identified as the groups discussed task 2b. There was one group who used the game to determine if we can see relativistic effects when we look at the Andromeda galaxy through a telescope. They calculated the β-factor of the Andromeda galaxy and estimated how many orbs they needed to collect to match the speed the galaxy has. They predicted that if we see clear differences in the game collecting a small number of orbs then we would also see relativistic effects looking at the Andromeda galaxy. The environment did not appear any different in the game at that speed and therefore, they conclude that they cannot see any relativistic effects looking at the Andromeda galaxy through a telescope.

> *G^9: Speed 0,3% of c $\Longrightarrow$ tiny relativistic effect. When a small amount of the orbs is collected, then the relativistic effects are not noticeable $\Longrightarrow$ no effects would be seen in the galaxy.*

The group did not present any screenshot of it but I have tested it (see figure 20). This way of using the game can be useful when a teacher is lecturing about special relativity. The teacher can illustrate that it requires speeds that are much faster than the one in figure 20 before we can notice the different colors in the sky as we look in different angles.

Figure 20. Test of the statement of G^9. In a) the player is at rest and at b) the player is traveling straight ahead after collecting 1 orb. The screenshots are taken at the same place. The orb at the center in a) is the one collected in b)

Chapter 5

5 Discussion & Conclusions

The project aimed to answer following questions:

- What relativistic visual effects do students report to have noticed while playing the educational computer game “A slower speed of light”?
- How do students relate (a) the observed visual effects and (b) values of physics quantities arrived at by applying Lorentz transformations?
- Can this computer game be used productively for physics teaching? If so, in what way and for what purpose?

5.1 Discussion of Research Question 1

We divide the relativistic effects into two categories: optical and kinematic effects. In table 1 it is clear that the optical effects were dominant in comparison to the kinematic effects. In the group submission, 32 effects were reported and ~ 69% of those \were optical. In the individual submission 105 effects were reported and ~ 68% of those were optical.

The dominant optical effect that the students reported that they noticed was the relativistic Doppler effect. An explanation could be that they noticed it before the other effects. The relativistic Doppler effect becomes clear already after 25 orbs (greener environment) were collected while the effects from relativistic kinematics are not clear until they have collected approximately 65 orbs. Before that, the searchlight effect becomes clear too which could explain why the optical effects were dominant in comparison to the kinematical effects. As the player slows down the speed of light further the visible light gets redshifted into the IR spectra which increases the difficulty to study the kinematic effects. It is also possible that they selected the relativistic Doppler effect because they needed to understand it in order to solve task 2a. There is also a lot of information about the relativistic Doppler effect online and in their collection of formulas (Nordling 2008), equation (24) can be found.

I also suspect that the students also used the credit section of the game to find information about the effects. There were submissions where they copied the credit section of the game. The credit section gave a short description of the effects that could be seen. It uses terms that is not scientific such as the runtime effect. The searchlight effect is scientific but not in course literature such as Rindler (1991). He calls the searchlight effect *relativistic aberration* where he also includes the distortions. The course literature that the students used (Thornton & Marion, 2004) does use those expressions either. They use the term *aberration of light* to explain relativistic aberration.

The most reported effect from relativistic kinematics was length contraction, despite most of the students and groups not describing a situation where they actually saw a contraction of an observed length, but rather referring to the apparent changing of the dimensions of object in the game more generally. In table 2, there are examples where students motivate their answer that is contradictory. We notice that it is harder for the students to describe their experience of kinematic effects in comparison to the optical. As they described how it was noticed they referred to the blue and redshifts but they could not find a situation of where the length contraction was seen.

5.2 Discussion of Research Question 2

It is important to be aware that the only preparation provided by the course was one lecture about the Lorentz transformation, length contraction and time dilation. As the students described how they saw the effects, there were cases where they could link what they see to values predicted by the Lorentz transformation. They could relate the blue and redshifts to the relativistic Doppler effect. At the seminar, they also showed that they could handle them quantitatively by solving task 2a. But the only case where they looked were in the direction of motion. Which is when the classic contribution to the Doppler effect was greatest. In task 2a, some groups added a comment about what wavelength we would see if the angle of the jets were 90 degrees relative to Earth.

> *G^{16}: When θ approaches $90°$, the wavelength $\lambda' \rightarrow \lambda''$ and be equal to $\lambda_0\gamma$*

G^{16} does not make a comparison with the non-relativistic Doppler effect. I speculate that the group just thought of this as a different value of the wavelength instead of the difference between non-relativistic and relativistic Doppler effect. It is an interesting finding because it indicates what the students expect teachers to perceive as relevant. This makes it more unlikely that the students understand how they can see the actual length contraction. When it comes to effects from relativistic kinematics it is more difficult for students to link what they see in the game to the predictions of the Lorentz transformation. It was explained by students in unconvincing terms. When students tried to explain how they saw the length contraction they did not present any screenshots of contracted lengths or pointed out a situation of where it was seen. We can actually see that some students and groups tried to "force the theory" onto their observations by referring to the 2nd postulate and the equations of length contraction:

> *S^6: We can see that length contraction occurs. It occurs because c is constant. To maintain that, the distance must change.*
>
> *S^8: Spacetime has to change because c is constant. An observer that travels fast will travel a shorter distance because of length contraction. The distance that the observer measures is*
> $L = L_0\sqrt{1 - v^2/c^2}$
>
> *S^{18}: Length contraction: As we move forward we experience that the distance to an object increases.*
>
> *G^5: $x' = \gamma(x - vt)$. If we travel away from the object $v > 0$ then $x' < x$, the distance to the object will be shorter. Which was seen when the environment*

appeared zoomed in. If we traveled towards the object $v < 0$, then $x' > x$. Which was seen when the environment was zoomed out

The students say they noticed length contraction because of changes in lengths and the only relativistic effect they know that is related to changes in lengths is length contraction. What most of the groups and students actually see in the game, is relativistic aberration and not length contraction. The relativistic aberration was not selected by many students but one reason could be because of the credit formulation of it:

"Light also behaves like a stream of particles called photons. When you run towards a stream of photons, more photons hit you and the object becomes brighter. This effect is also known as Relativistic Aberration."

From the perspective of a student that do not have much experience of relativity, the student is likely to interpret relativistic aberration being the same as intensity. The searchlight effect is a consequence of the relativistic aberration but it should emphasize that relativistic aberration is wider than this optical effect. It also the reason to why distances to objects appear to increase as you travel towards them at a relativistic speed (see figure 10b).

From the extract about students' perception about length contraction they tend to study cases where they look in the direction of motion. Note that they are not incorrect when they say that length contraction is there but it is not what they are describing. S[18] described that distances to objects increases as the they travel towards them. G[5] tries to manipulate the formulas. This is an interesting *finding* because these statements illustrate what the students expect to see before they played the game. They expected that the visual effects should match with the predictions of the Lorentz transformation. That is a possible explanation of why they tried to force the theory onto what they saw in the game. It also implies that it is difficult for the students to make a distinction between an effect that is being observed and when an effect is seen. However, their solutions also indicate that they notice that what they see is different from the predictions of the Lorentz transformation. If they did not realize that then the submissions would have contained more screenshots or situations of where they state that they saw it. A consequence that follows is that the students do not understand what they need to do to see the actual length contraction. They have to look perpendicular to their direction of motion

Another indication of students struggling making a distinction between see by naked eye and observation could be seen in how they solved task 2b. Most of the students and groups looked for data about if the galaxy has a velocity relative to earth at all and inserted it into formulas instead of determining if it is relativistic. They calculated a very small blue shift and concluded that they would be able to see this. That is not how they see the relativistic Doppler effect. See the relativistic Doppler shift by naked eye means that the disk of the galaxy shifts colors as you look closer to the edges of it. In order to see that, taking the game as a reference, 64 orbs (see figure 21) needed to be collected. According to Hodge (1992) the Andromeda galaxy is approaching the sun at 310 km/s which is 0,001c. It is a small difference even if the angle, $\theta = 0$. Using (24), the observed wavelength $\lambda_{obs} = 0.999\lambda_0$ where λ_0 the proper wavelength. Relativistic effects cannot be seen by looking through a telescope. In order to see those small effects other methods are required and it is somewhat surprising that the students did not think about that after solving task 2a.

Figure 21. Illustration of when the different colors is seen depending on the angle

5.3 Discussion of Research Question 3

From findings, we can see that game was used by only a few groups to answer different tasks than the first one. However, there was an interesting case of how G^9 used the game to evaluate if relativistic effects would be visible looking at the Andromeda galaxy. They calculated the β - factor of the Andromeda galaxy and predicted that if the effects are visible then collecting a small number of orbs would make clear changes in the game environment. That could be useful discussing motions of stars and galaxies e.g. what would Jupiter look like if it moves at 0,7c relative to Earth or if we observe a jet from an active galaxy nucleus, what does it look like? Furthermore, we could see that it was easier for the students to describe the optical effects than the effects from kinematics. I speculate that only an introduction to the Lorentz transformation is not enough for the students' to be able to work out the visual effects by themselves, a hand in problem of this kind would require more background for most of them. There were cases in my data where it was enough, but for the majority it was not.

However, if a teacher wants to use the game "A slower speed of light" for teaching, it is recommended that its purpose is to show that there are certain effects where observations and what we actually see is different. From the findings, I would say that the game can be appropriate for teaching if implemented in an appropriate way. I recommend that the assignment contains tasks that makes the students study different angles than their direction of motion. Then they get an opportunity to see the length contraction and discuss why appears in this particular situation. The implementation also depends on what kind of students the teacher has. In this project, the only preparation from the course was one lecture and from findings, that was not enough. The students needed an introduction to the differences between how they see optical effects and distortions before playing the game. The introductory does not need to be an entire course but a short discussion of what see an effect means and how it is different from observations. After that, the teacher could construct a hand in problem of similar structure as the one used in this project. However, if the students are taking an advanced course in special relativity, then an introduction may not be equally important. But if the students are high school students, then such use of the game is more likely to cause confusions. In a high-school context, the teacher could encourage the students to use the game as G^9 did. They used the game to determine if the relativistic effects would be visible. High-school students could use the game to explore when a speed results in visual effects. Otherwise, my recommendation is that the teacher uses the game as a demonstration of what a relativistic environment actually looks like. If used for a hand in problem it is recommended that the users use the game for more than one task. In findings, most of the students used the game in task one where they were directly instructed to use it. Yet, it could be useful in the remaining tasks. It is possible that those ways of using the game was unclear to most of the students. Therefore, it is recommended that the game is used in more than one task and that there are concrete hints accompanying each task on how to use it.

5.4 Limitations of this study

There is no such thing as no uncertainty. Research of visualization in special relativity is limited which complicates comparisons between findings. The existent studies did not use “A slower speed of light”, they used “Real time Relativity”. Likewise, it becomes complicated to discuss how valid the findings are. Also, the project has no control group. The findings are only valid to the circumstances of *this* study. Because of that, I do not suggest that this study will have the same outcome if repeated in different circumstances.

This study only used *one class*, this study is therefore limited to their experiences at *that* university. By modifying the students’ experiences e.g. more preparation about special relativity, more experience of modern physics courses such as quantum mechanics or making the students interact with the simulations for a longer time the findings could be different. The findings could also be different if repeated at a larger scale. Relative to the study of McGrath (2010) and Savage (2007) this study had fewer students and was only tested at one class. In their case it was also tested at several institutions. However, the study does increase the possibility for a teacher to understand challenges of applying the game in teaching e.g. making the students understand that see and observe sometimes does not match. The study also increases the possibility that the teacher plans his /her teaching more effectively e.g. by encouraging students to look in different directions than straight ahead. In order to do that successfully that teacher must reflect upon if his/her students are similar to those in this study. If no, then the teacher must correlate that.

5.5 Recommendations for education

The game “A slower speed of light” has potential to develop students' intuition about special relativity which was the purpose of the game (Kortemeyer 2013). The game gives them a *qualitative* insight into what a relativistic environment would look like. The next step is to study if the game also can develop a *quantitative* intuition about the visual effects that is not optical (i.e. beyond Doppler effect and searchlight effect). The students were more comfortable with the optical effects than the kinematic effects. In order to further develop relevant understanding of visual effects in special relativity, the game must give students better opportunities to perform their own experiments. In its current form, it is difficult to do experiments e.g. moving at certain angles which in this project could have been useful in task 2a. It could give the students an opportunity to actually see what the jets would look like. It is also difficult to estimate the players velocity which makes it more difficult to test predictions. In the game, the speed of the player can be adjusted and depending on how many orbs that are collected, the stronger the visual effects appear. But from a laboratory perspective, it would be more efficient if the player could see its walking speed in terms of the speed of light instead speed meter that does not use a linear scale. It is difficult to use the method as I did by estimating angles as it gives uncertainties. If the game showed what speed the player has it would be easier to quantitatively use the game. Another suggestion is that the game provides the opportunity to control how the ghost characters in the game environment should move, and also make all characters hold a twinkling flashlight. At rest, let them all be synchronized. If anyone would try to do a research project studying the suggestions given above it is recommended that the hand in problem (if used for data collection) is given to students who have taken an advanced course in special relativity. As mentioned before, in this project the only preparation the students were provided from the course was one lecture that emphasized the Lorentz transformation. And as we could see in the findings it is more difficult for the students to explain that than the optical effects. I also recommend making it possible to turn off the optical effects because as the player collect many orbs the relativistic Doppler effect shifts visible light into the IR spectra. In figure 6 we cannot see the left half because the light is shifted to invisible spectrums. That makes it more complicated to study effects such as length contraction.

If the suggestions above were implemented, there would be more possibilities of using the game in high school too. It invites students to directly study the time dilation which is relevant in both high school physics and university. While moving, the player would see that that their own flashlight twinkles more frequently than the ghosts'. That phenomena could be an introduction to study simultaneity which is an important concept in special relativity. However, in the current state of the game it can be used to illustrate what kind of speeds is needed to actually see the relativistic effects. For demonstration, I recommend a teacher to use it in a similar as G^9 did in task 2b where they used data to calculate the β-factor and collected a small number of orbs and study if the environment changes. If these points were implemented, this could be a powerful tool that both university and high school teachers could use to an even greater degree. But if applied for high school it would be convenient to turn off the optical effects since they study special relativity before waves and optics (Sweden). In a university context, it is usually the opposite. According to Skolverket (2010) (National Agency for Education in Sweden), time dilation is also part of the central content of the course Physics 1, while relativistic Doppler effect is not.

5.6 Conclusion

This project studied how visualization of relativistic effects is perceived by students. From findings, there is evidence that the optical effects are selected more frequently than the kinematic effects. We have found evidence that the students first expected the visual effects to match with the predictions from the Lorentz transformation. This implies that in the context of special relativity, students struggle making a distinction between effects we observe and those that we see. That could be seen by comparing how they described the relativistic Doppler effect and length contraction. If a teacher wants to use the game "A slower speed of light" for teaching, I suggest considering the following points:

i) Use the game, if the intention is that the students shall see that visual effects do not match with predictions of the Lorentz transformation

ii) Think of who your target is. Are they bachelor/master students? Give the students an introductory to necessary concepts, e.g. what *see* means.

iii) Make sure that the game can be applied in different kinds of tasks (and give hints on how to do it).

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8 Appendix

Appendix A

Problem set 1: Relativistic optics

1. Download and play *A slower speed of light*. Pick one of the visual effects of special relativity that you see in the game and give an explanation for it. Please explain it as you would if you were to explain it to a fellow student. Include formulas when appropriate in your description.

Figure 1: The game A slower speed of light.

2. A galaxy with an active nucleus is observed from earth. The nucleus has emitted two jets of equal speed v in opposite directions and each at an angle θ to the direction to earth. In the light emitted from the jets a spectral line from singly ionised Magnesium is identified. The proper wave length of the spectral line is $\lambda_0 = 448.1$ nm. The spectral lines from the two jets are measured at $\lambda_+ = 420.2$ nm and $\lambda_- = 700.1$ nm.

   You may assume that the velocity of the galaxy itself is negligible.

   (a) Find the measured wavelength in terms of the angle θ, the velocity v and the proper wave length λ_0.

   (b) Find the angle θ and the velocity v of the jets emitted.

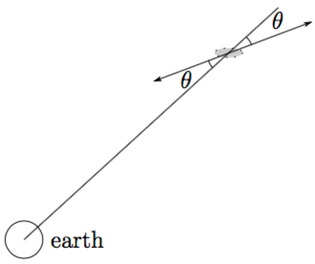
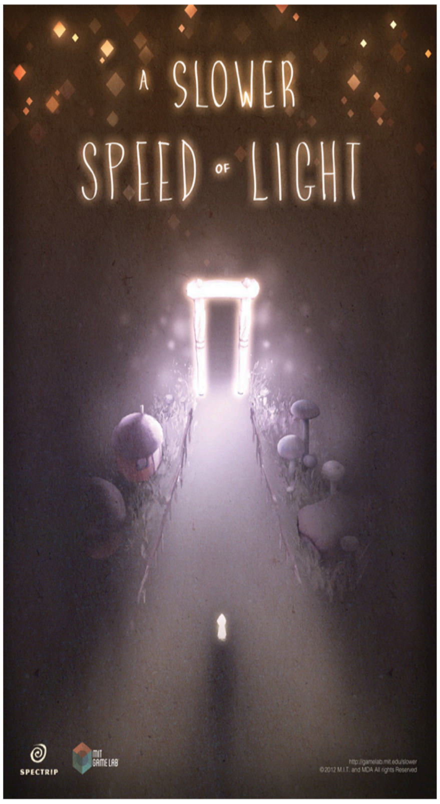


Figure 2: A galaxy with an active nucleus emits two jets of equal speed.

Deadline: September 18, 2015

Appendix B

1. Ladda ner och spela spelet [A slower speed of light]. Välj ut två av effekterna som spelet illustrerar och förklara fysiken bakom dessa. Använd gärna screenshots från spelet i din förklaring. Dina kursare är målgruppen för din förklaring.



2. a) Om du har samlat tillräckligt många *orbs* i spelet och ställer dig still i närheten av figurerna som rör sig mellan hyddorna så kan du observera att de ser olika ut beroende på hur de rör sig i förhållande till dig. Samma effekt spelar roll i observationell astronomi vilket illustreras av följande exempel.

   En galax med en aktiv kärna observeras från jorden. Från galaxen sänds jetstrålar av plasma ut. Jetstrålarna sänds ut i motsatta riktningar från galaxen, med samma fart och i vissa riktningar relativt jorden. I det ljus som observeras på jorden identifieras en spektrallinje från magnesium, denna linje motsvarar en våglängd på $\lambda_0 = 448.1$ nm i vilosystemet. I ljuset från den ena strålen är denna spektrallinje förskjuten till en våglängd på 420.2 nm och i den andra till en våglängd på 700.1 nm.

   Vilka slutsatser kan du dra om hastigheten hos de två strålarna? Bestäm strålarnas riktning relativt jorden. Hur skulle observationerna av våglängder påverkas om strålarnas riktning relativt jorden skulle ändras? Det går bra att anta att själva galaxen är i vila relativt jorden.

   b) Man kan med en bra kamera ta foton av Andromedagalaxen från jorden, ännu bättre blir det
   om man har tillgång till ett teleskop. Detta beskrivs exempelvis [här].
   När man gör det, ser man då någon relativistisk effekt? Resonera med hjälp av dina tidigare resultat och dina erfarenheter från spelet.

Appendix C

We will now derive the formula for relativistic aberration. First, we need an expression for relativistic velocity. Suppose an object is traveling at speed v along the x-axis. We can obtain its velocity by taking the derivative of equation (3):

$$u'_x = \frac{dx'}{dt'} = \frac{d\big(\gamma(x - vt)\big)}{d\left(\gamma\left(t - \frac{vx}{c^2}\right)\right)} = \frac{d(x - vt)}{d\left(t - \frac{vx}{c^2}\right)} = \frac{\left(\frac{dx}{dt}\right) - v\left(\frac{dt}{dt}\right)}{\left(\frac{dt}{dt}\right) - \frac{v}{c^2}\left(\frac{dx}{dt}\right)} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

Since v is in the x-direction, the y and z-coordinate are unchanged by the Lorentz transformation (y'= y) and (z'= z). The velocity transformation in the y-direction is following:

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{d\left(\gamma\left(t - \frac{vx}{c^2}\right)\right)} = \frac{\frac{dy}{dt}}{\gamma\left(\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}\right)} = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

Now suppose we have the same situation as in figure 13. S' is emitting a photon to S. To S, it appears to be emitted from an angle θ and θ' to S'. We can describe the angles in terms velocities as following.

$$c = -u_x \cos(\theta) \quad c = -u'_x \cos(\theta')$$

If we set the equations equal to each other we get:

$$\cos(\theta') = \frac{u_x \cos(\theta)}{u'_x} = \frac{u_x \cos(\theta)\left(1 - \frac{vu_x}{c^2}\right)}{(u_x - v)} = \frac{\cos(\theta)\left(1 - \frac{vu_x}{c^2}\right)}{\left(1 - \frac{v}{u_x}\right)} \rightarrow$$

$$\cos(\theta') = \frac{\cos(\theta)\left(1 + \frac{v}{c}\frac{1}{\cos(\theta)}\right)}{\left(1 + \frac{v}{c}\cos(\theta)\right)} = \frac{\cos(\theta) + \beta}{1 + \beta\cos(\theta)}$$

This is the case when S' emitted a photon to S. However, if the opposite happens, we get:

$$\cos(\theta') = \frac{\cos(\theta) - \beta}{1 - \beta\cos(\theta)}$$

An alternate way of writing this formula can be done by applying the trigonometric identity:

$$\tan\left(\frac{\theta'}{2}\right) = \frac{\sin(\theta')}{1 + \cos(\theta')}$$

This gives (32):

$$\tan\left(\frac{\theta'}{2}\right) = \sqrt{\frac{c - v}{c + v}}\tan\left(\frac{\theta}{2}\right)$$