other hand, these possibilities may not all be widely known, and for this reason we have thought it worthwhile to summarize them briefly.

ACKNOWLEDGMENTS

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The Geometrical Appearance of Large Objects Moving at Relativistic Speeds

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The calculated geometrical appearance of objects moving at relativistic speeds and subtending large angles at the observer is illustrated by diagrams of a plane grid and perspective views of a group of boxes. In addition to the distortion of scales in the direction of motion, planes perpendicular to the motion appear as hyperboloids. Contrary to an impression which might be taken from some papers on the subject, the Lorentz contraction *is* visible under suitable conditions, in particular for observations approximately at right angles to the motion.

THE discussion of results in special relativity requires great care if misinterpretations are to be avoided. There is no more cogent example of this fact than the case of the predicted appearance of objects moving at relativistic speeds.[1–3] As Weisskopf[4] has pointed out, until 1959 most physicists tacitly assumed that the Lorentz contraction could *in general* be seen or photographed at sufficiently high speeds. The result that a rest length L_0 will be measured as contracted to $L_0(1-v^2/c^2)^{\frac{1}{2}}$ when moving at speed v, was described as "the length will be observed as contracted" and this in turn as "the length will be seen as contracted." Thus by slight successive changes in wording, statements which were quite correct for one type of observation, *viz. measurement,* became quite wrong for another, *viz. visual observation.* In part, the difficulty arose because the "classical" time-of-transmission effects were overlooked in favor of the more intriguing relativistic effects.

The papers both of Terrel[2] and of Weisskopf[4] deal with the case of objects which subtend a small angle at the observer. Under such conditions the appearance can be related to a mere rotation of the object. With larger objects it was recognized that distortion of the shape would occur. Boas[5] has discussed how straight lines generally appear curved, but that, as shown first by Penrose,[1] a sphere always presents a circular disk.

Cases of rectangular objects subtending large angles at the observer have been discussed by Atwater[6] and by Yngstrom.[7] Atwater compares the simultaneous views of a rectangle by two coincident observers one of which is at rest relative to the rectangle while the other is in motion. Yngstrom shows simultaneous pictures of a cube taken by three coincident cameras in different states of motion.

To illustrate the nature of the visible distortion of moving objects we have compared the appearance of rectangular shapes when in motion with their appearance at rest. In the work of Atwater and of Yngstrom the coincident but differently moving observers taking simultaneous views find the object in *different* directions. For our cases one observer compares the views of an object at rest and in motion when the object, or at least a central point of it, is in the *same* direction.

All the effects can be determined from the relativistic aberration formula[7] but it is often simpler to use directly the Lorentz transforma-

[1] R. Penrose, Proc. Cambridge Phil. Soc. **55**, 137 (1959).
[2] J. Terrel, Phys. Rev. **116**, 1041 (1959).
[3] R. Weinstein, Am. J. Phys. **28**, 607 (1960).
[4] V. F. Weisskopf, Phys. Today **13**, No. 9, 24 (1960).
[5] Mary L. Boas, Am. J. Phys. **29**, 283 (1961).
[6] H. A. Atwater, J. Opt. Soc. Am. **52**, 184 (1962).
[7] S. Yngstrom, Arkiv Fysik **23**, 367 (1962).

tions and the time of transmission of light signals as described in the Appendix.

Figure 1 shows the appearance of a plane grid moving at different speeds when observed from a point unit distance in front of the grid. The appearance or picture can be defined here by the intersection with the plane of the grid of light rays which arrive simultaneously at the observer. The pictures can also be imagined as taken by a small, very wide-angle camera with the axis of its lens perpendicular to the plane of the grid. The motion is to the right and the picture is taken so that the origin is, in each case, at the centre of the field. The main features are (i) the curving of the vertical lines which become hyperbolas (see Appendix), (ii) the contraction of horizontal lengths to the right, i.e., as they recede, and (iii) the extension of lengths to the left, i.e., as they approach (see Ref. 3). Submerged within these features is the Lorentz contraction which can be clearly distinguished near the origin where the angle of observation is about 90° to the direction of motion.

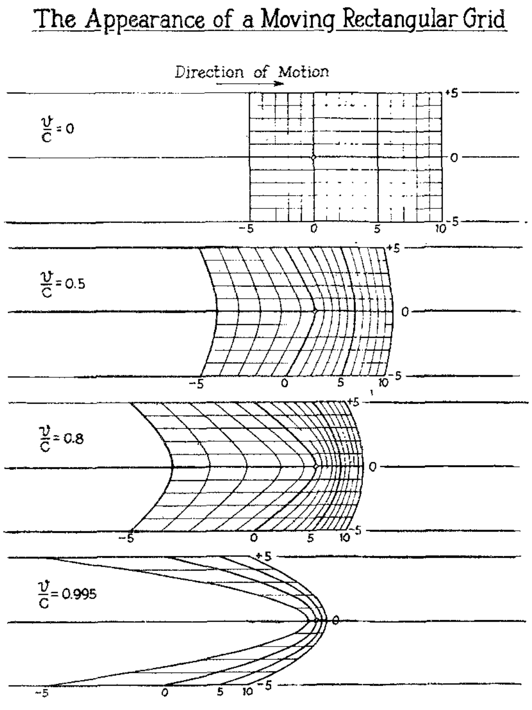

FIG. 1. The appearance of a plane grid moving at relativistic speeds. The observer is unit distance in front of the origin. For each view the direction in which the observer sees the origin is perpendicular to the motion.

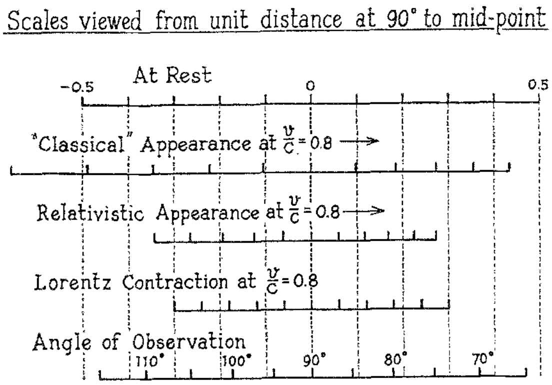

FIG. 2. Comparison of the "classical" and the relativistic appearance of a uniform scale in motion.

An enlargement of the grid scale along the horizontal line near the origin for $v/c = 0.8$ is given in Fig. 2. The scale is imagined as viewed from a location unit distance from its midpoint. The drawing shows (i) the scale at rest; (ii) the "classical" appearance when it is moving to the right at $v/c = 0.8$ (the "ether" is assumed at rest with respect to the observer and the appearance depends on the time of transmission of light); (iii) the relativistic appearance at $v/c = 0.8$; (iv) the pure Lorentz contraction at $v/c = 0.8$; (v) the angular position in the observer's field of view. It is evident that as the angle of observation approaches 90°, the "classical" appearance becomes that of the scale at rest, whereas the relativistic appearance reduces to the Lorentz contraction.

It is worth noting that in the papers referred to, even as the authors[2,4] correct some misunderstandings they present viewpoints which could lead to new ones. The title of Terrel's paper "The Invisibility of the Lorentz Contraction" and the implied denial of Lorentz's original statement about photographing the contraction, could be misleading. When a rod is moving parallel to its length and is seen or photographed at right angles, the Lorentz contraction *will be observed* relative say to an adjacent rod which is at rest. If the orientation of the rod is known, or is controlled, or verified by the position of adjacent rest objects (e.g., moves in a track), or if the rod is long and marked off as a scale, then the explanation of the appearance in terms of a rotation of the object is quite inadequate. As illustrated by Fig. 2, the Lorentz contraction *is* visi-

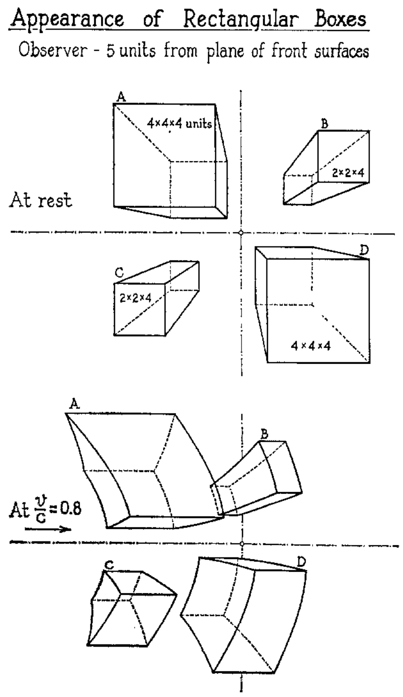

FIG. 3. The perspective views of rectangular boxes at rest and in relativistic motion. In each case the origin is seen in a direction at right angles to the front plane of the boxes and thus at right angles to the motion.

ble, although primarily near 90° where the "classical" effects are negligible.

The perspective views of four rectangular boxes at rest and in motion are given in Fig. 3. The observer is 5 units from the near plane of the boxes and looking in the direction of the origin. The grossly altered appearance of the moving boxes might be described as a nonuniform shear deformation in each of the two perpendicular planes through the line of motion. The effects described as an apparent rotation when the object subtends a small angle are evident. The distortion of the vertical edges can readily be related to the hyperbolas of Fig. 1. Horizontal edges in the direction of motion remain straight; those perpendicular to the motion become hyperbolas which in the drawing are altered by perspective.

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APPENDIX

The Apparent Positions of Points in a Moving Coordinate System

The observer is at rest in a coordinate system S and on the z axis at a distance d from the origin. The system S' is moving in the x direction with a velocity β in terms of c. The x axes are in the same line and when $t=t'=0$ the two origins 0 and 0′ are coincident. For 0′ to appear at 0, the instant of observation (i.e., photographing) is at $t=d/c$ and the light from 0′ will have been emitted at $t=0$. The light from any other point $P'(x',y',z')$ in S' which appears at $P(x,y,z)$ in S must have been emitted at $t=-[(x^2+y^2+(z-d)^2)^{\frac{1}{2}}-d]/c$ in order to be observed simultaneously with the light from 0′.

The Lorentz transformations give

$$x'=\gamma(x-\beta ct)=\gamma\{x+\beta[(x^2+y^2+(z-d)^2)^{\frac{1}{2}}-d]\},$$
$$y'=y,\quad z'=z,$$

where as usual $\gamma^{-2}=1-\beta^2$.

By expressing x, y, z in terms of x', y', z' the positions of points in S' can be found as they appear to the observer in S, *viz.*

$$x=\gamma\{(x'+\gamma\beta d)-\beta[(x'+\gamma\beta d)^2+y'^2+(z'-d)^2]^{\frac{1}{2}}\},$$
$$y=y',\quad z=z'.$$

Alternatively it can be noted that the event of observation of light from the 00′ coincidence is $(0,0,d,d/c)$ in S or $(-\gamma\beta d,\ 0,\ d,\ \gamma d/c)$ in S'. Light from a point $(x'y'z')$ on the object which arrives simultaneously with the ray from 00′ must have started at

$$t'=-\{[(x'+\gamma\beta d)^2+y'^2+(z'-d)^2]^{\frac{1}{2}}-\gamma d\}/c.$$

Then the above expression for x follows directly from the Lorentz transformation $x=\gamma(x'+vt')$.

The apparent shape of a y'–z' plane is one branch of a two-sheeted hyperboloid of revolution as noted by Yngstrom.[7] The shape is obtained by setting $x'=a$ constant or, say, $(x'+\gamma\beta d)=A$. Then the above relation gives

$$(x-\gamma A)^2/\gamma^2\beta^2A^2-y^2/A^2-(z-d)^2/A^2=1,$$

which, it can be seen, is the equation of an hyperboloid. Hence, straight lines perpendicular to the x' axis appear hyperbolic in shape.