A Carom Billiard to Understand Special Relativity

Tony Doat*
VENISE team, CNRS-LIMSI
Orsay, France

Etienne Parizot†
APC, Paris Diderot University
Paris, France

Jean-Marc Vézien‡
VENISE team, CNRS-LIMSI
Orsay, France

Abstract

Relativity, as introduced by *Einstein*, is regarded as one of the most important revolutions in the history of physics. Nevertheless, the observation of direct outcomes of this theory on mundane objects is impossible because they can only be witnessed when travelling at relative speeds approaching the velocity of light c. These effects are so counterintuitive and contradicting with our daily understanding of space and time that physics students find it hard to learn relativity beyond mathematical equations and to understand the deep implications of the theory.

Advances in Computer Graphics and Interaction Technology now make it possible to actually experiment the effects of relativity in a 3D immersive environment. We propose a Virtual Reality framework to study and learn relativity as well as to develop some intuition of the relativistic effects and the quadri-dimensional reality of space-time. More precisely, an innovative rendering engine and a non-Newtonian physics engine are combined to compute relativistic effects. Experiments are proposed within a game-oriented carom billiard environment. Our work improves over previous efforts in the ability i) to render in real-time multiple relativistic objects, each moving with a different velocity vector ii) to enable interactions between objects and iii) to enable the user to interact with the objects and modify the scene dynamically.

The originality of our 4D rendering engine lies in its capacity to efficiently store and retrieve non-simultaneous past space-time events that are visible by an observer at a specific location and at a given instant of his proper time.

Index Terms: I.3.3 [Computer Graphics]: Picture/Image Generation—Viewing algorithms I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Physically based modeling K.3.2 [Computer Graphics]: Computer and Information Science Education—Computer science education

1 Relativity understanding through simulation

Learning physical sciences often requires imagination and a solid ability for abstraction. This is particularly true in the case of the theory of Relativity even in the special case, which modifies our basic intuition about space and time.

What the theory of Relativity teaches us, however, is that neither space and time are absolute. Instead, they make up a global geometric structure with 4 dimensions, called space-time, whose "time" and "space" components depend on the reference frame used to describe physical bodies and events in terms of positions and instants. In particular, the *length* of an given object, as well as the *duration* of a given phenomenon (between two well-defined events) will not only appear different, but actually *be* different for two observers moving with respect to one another.

As it turns out, this has some very deep consequences about the nature of space and time, that are in direct contradiction with our intuition. As ordinary human experience is limited to very slow velocities compared to the speed of light, c, it leads to a number of physical effects that are particularly puzzling to the student who first discovers them, and even to experienced physicists. This basic limitation, however, is challenged and actually overcome by modern information technology and simulation systems: what about creating a world with an easily accessible speed of light?

As in many other fields, Computer Graphics(CG) and Virtual Reality(VR) provide significant benefits to help someone to grasp complex, unfamiliar concepts. In the case of Relativity, CG can visually recreate physically realistic phenomena, while VR enables scientists to immerse users into a virtual world where very fast motions take place. In VR, one can interact with 4D phenomena and travel at the speed of light, or equivalently reduce the speed of light to a few centimetres per second.

This perspective triggered the creation of an interdisciplinary team, gathering VR, physicists and didacticians, with the aim of merging advanced 3D graphics with VR interfaces in order to create an appropriate environment to study Relativity as well as to develop some intuition of the quadri-dimensional reality of space-time.

2 Virtual relativity: challenges and issues

Several researchers [7, 1] have studied the problem of how students learn relativity concepts, including the invariance of the speed of light, the concepts of frame of reference and the most disturbing problems: the relativity of simultaneity of events. It was found that even Master students do not properly grasp the fundamentals and consequences of relativistic physics. Hence there is obvious interest in using modern computer simulation to help them to understand how relativity shapes modern physics.

Very early, mathematics were used to predict what objects would look like in relativistic motions. The first solutions appeared in 1959 with *Penrose* [4]. Then, researchers focused on various relativistic effects (such as Doppler, search light, aberration of light, etc.). For real-time rendering, two main techniques were proposed: the ray-tracing technique [3] and the polygon rendering technique [2]. *Weiskopf* [8] extended the classical rendering techniques and presented a solution exploiting modern graphic cards (GPU) for special relativity. Recently, *Savage* [6] developed a game-like computer simulation where the user can fly at relativistic speeds and see the world according to his reference frame in real-time. However, all these techniques only handle one moving object relative to a static world. No interaction is possible to modify the scene and appreciate the resulting effects on the simulation.

Our approach consists in creating an application that i) handles 4D phenomena according to the laws of special relativity, ii) renders the phenomena as they appear visually, with no restriction on the velocity of the observer and/or the objects in the scene and iii) enables the user to interact with objects and to influence the scene. As a first implementation, we propose an application that is easy to use and is likely to awake curiosity about a 4D world: a relativistic carom billiard. The French billiard is close to the snooker game in essence but has fewer balls and no holes. This application enables the users to observe and interact with several independent objects, featuring relativistic motions and collisions, and to apprehend the notion of *event*, which is a key concept for the didactic field.

*e-mail: tony.doat@limsi.fr
†e-mail: parizot@apc.univ-paris7.fr
‡e-mail: jean-marc.vezien@limsi.fr

IEEE Virtual Reality 2011
19 - 23 March, Singapore
978-1-4577-0038-5/11/$26.00 ©2011 IEEE

3 Rendering

To simulate and render an accurate relativistic spacetime, two key features of the theory of Special Relativity have a direct impact. First, the lengths, the sizes and the durations are not invariant and depend on the relative velocity of the objects and reference frames involved. Second, the speed of light, c, is finite (and invariant), so we do not see the objects where they are *now*, but where they were when they emitted the photons that we perceive now.

Both of the above-mentioned effects – the modification of the objects geometry and the time delay between the events and their perception – combine in a non-trivial way to give rise to images where the objects may be distorted compared to what would be obtained by classical rendering.

3.1 Implementing the Lorentzian contraction of lengths

In Computer Graphics, each object is generally discretized with an appropriate number of points. This definition of objects, however, may be problematic in a relativistic context, because this intrinsic definition specifies the relative distances between the points *in the rest frame of the object*. Now, a consequence of Relativity is that the length between two points depends on the reference frame, i.e. we have to place ourselves in a given reference frame to follow the evolution of a simulation. In our case, the rest frame of the billiard board appears to be the obvious choice. The so-called Lorentz transform, specifies how coordinates transform from one rest frame to another. The net effect of the movement of an object is an effective contraction of all distances along the direction of the motion, with a factor, γ, called the *Lorentz factor*, defined as $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, where v is the velocity of the object in the frame of reference under consideration, and c is the speed of light. In the billiard simulation, an intrinsically spherical ball moving with Lorentz factor γ with respect to the board frame becomes an ellipsoid with a smaller axis equal to the radius of the sphere divided by γ oriented in the direction of motion. Faster moving balls are thus more "compressed". Once the warped scene coordinates are obtained in the observer's reference frame, the rendering process can take place to compute the images he actually perceives.

3.2 Exploiting the invariance of space-time intervals

To calculate physically correct images, we have to determine what a given observer sees at a given location at a given time (i.e. at a given point in spacetime). This requires a framework in which the whole history of the scene is accessible to allow the exploration of the past positions of the various elements of the scene. Thanks to this history, the challenge is to determine the *emission event* of the photons reaching the eyes of the observer at the observation event.

Thus, the determination of these emission events is simple and fast, if one uses the space-time interval between two events, defined as $\delta s^2 = c^2\delta t^2 - \delta l^2$, in time-like convention. Here δt and δl are respectively the time interval and the (spatial) distance between the events. Both δt and δl depend on the reference frame, but the central feature of Relativity is that δs does not: it is invariant.

Now the idea behind our algorithm is that photons propagate along paths with null space-time intervals, so the emission event, $(t_e, \vec{r}_e)$, and the observation event, $(t_o, \vec{r}_o)$, are always related by the simplest equation: $\Delta s^2 = 0$. Then, for the current observation event, we search the emission event that satisfies $\Delta s([\text{Emission}], [\text{Observation}]) = 0$. Causality further ensures that this event is unique. Photons emitted at instants for which $\Delta s < 0$ have not yet reached the observer, while those emitted at instants for which $\Delta s > 0$ have already struck him in the past.

The use of a dichotomy procedure to solve the above equation for each part of each object of the scene enables real-time simulation, even for quite elaborated scenes. Besides, since the space-time interval is an invariant, the technique can be applied identically in any reference frame. It ensures a consistent approach for rendering, whatever the number of moving objects and observers.

4 Conclusion

We have presented an application that simulates a relativistic carom billiard, in which the speed of light may be reduced to such a velocity that unfamiliar or even counterintuitive effects can be directly experienced. This tool significantly improves over previous attempts to simulate four-dimensional, relativistic space time, by allowing the user not only to observe an otherwise static environment while moving at relativistic speed, but also to interact with objects, and explore a consistent 4D world in which different objects with arbitrary velocities and motions can be simulated and followed simultaneously by several observers.

This simulator also highlights some basic differences between the classical and relativistic cases, from the kinematics and dynamics points of view. We believe that the ability for the user to discover these effects by himself, through a direct exploration of and interaction with a relativistic 4D scene that is physically consistent, is of great didactic value. Developing a more intuitive understanding of Relativity, through experience, should also be valuable for experienced physicists. Initial polls were already conducted on physics students to identify common misconceptions and problems encountered while learning Relativity. We are now in the process of designing VR experiments based on these findings [5]. It is foreseen that exams taken after these tests will show a significant increase of understanding and performance.

More generally, we intend to use the strengths of Virtual Reality and Computer Graphics to further enhance the interactive experience of a relativistic world which is, after all, our world, even though we never experience its true nature because the speed of light is so large compared to the relative velocities we usually achieve. Forthcoming developments of our approach will involve the use of haptics, to interact with the environment in a more physical way, experiencing some aspects of relativistic dynamics. Also, the evaluation of our relativistic applications in a large-scale, immersive VR environment is underway.

Acknowledgements

We thank *Cécile De Hosson* and *Isabelle Kermen* for their helpful advice. This research was made possible by grant *BLAN*08 – 3_314843 of the *French National Research Agency*.

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